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THEORY OF PLANE, SYMMETRICAL INTAKE DIFFUSERS

By Walter Brödel

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PART I

I - THE THREE-PARAMETRICAL GROUP OF THE INTAKE DIFFUSERS

1 - Introduction

The present report ties in with the investigations on inlet diffusers by P. Ruden. The theory developed by Ruden (reference 1) had produced results which found excellent confirmation in wind-tunnel tests (reference 2) and in spite of certain still-existing defects, are technically very promising. The reasons for the new theory of the diffuser forms indicated by Ruden are twofold: first, the arguments adduced in reference 1 deal only with one specific operating condition, that is, a certain ratio of mean velocity within the diffuser to flying speed, while in the present report any desired velocity ratios are involved; second, a different choice of parameters and the increased possibilities of variations result in diffuser forms which cannot be reconciled at once with Ruden's theory. The first enables a theoretical check of the measurements made with Ruden's diffusers at variable velocity ratio, the second permits the calculation of diffuser types which in many respects are superior to Ruden's diffusers.

So, while the present report seems to be a supplement and continuation of Ruden's report (reference 1), it is nevertheless a study by itself and does not rest on the previous knowledge of Ruden's theory, with exception of the first part of section 6. The aerodynamic problem involved is the following (reference 1, page 3):

A certain volume of air from the air stream is to be intercepted and by conversion of speed to pressure or pressure into speed to be conveyed to certain airplane accessory devices. The manner in which the ratio of the velocity inside the diffuser to flying speed $\frac{w_i}{w_\infty}$ is regulated is not involved here; the most important thing is to find the best possible

*"Zur Theorie ebner, symmetrischer Fangdiffusoren." Zentrale für wissenschaftliches Berichtswesen der Luftfahrtforschung des Generalluftzeugmeisters (ZWB), Berlin-Adlershof, Forschungsbericht Nr. 1475/1 and 2, September 20, 1941.

shape for the contour of the nose. This, on the other hand, postulates that the usually inevitable increases of speed be as low as possible, and also that the wall thickness of the device be a minimum. It is found that these two requirements are in a certain contrast to each other; generally speaking, a reduction of the increase of speed at the diffusers in question can be attained only by an increase in wall thickness. One of the principal tasks is to analyze this relationship.

In conformity with reference 1, the following idealizations are effected: The flow is assumed to be incompressible and perfect, so that it can be represented in the conventional manner by analytical functions of a complex variable; the diffuser is assumed to extend to infinity in flow direction.

2 - Hydrodynamic Mapping And Introduction of the Parallel Strip as Reference Area

Figure 1 represents the contours of a plane, symmetrical inlet diffuser and at the same time indicates the qualitative performance of a corresponding flow. The problem is to change over from this schematic figure to a quantitative one. This is accomplished by the method of hydrodynamic mapping.

Hydrodynamic mapping is, as is known, conformal mapping in which one space, along with the flow in that space, is transformed into another. Three functions are involved in the analytical representation: The stream function (the complex stream potential) $F(z)$ of the original space, the mapping function $\zeta(z)$ which is considered to take a space on the z-plane to a space on the ζ-plane, and the stream function $\Phi(\zeta)$ of the transformed region; $F(z) = \Phi(\zeta)$, that is, the values of the stream function are simply transplanted from the z- to the ζ-plane. The "complex velocities" $w(z)$ and $\omega(\zeta)$ are given by $w = \frac{dF}{dz}$ and $\omega = \frac{d\Phi}{d\zeta}$ and related accordingly by

$$w \, dz = \omega \, d\zeta$$

or (1)

$$w = \omega \Big/ \frac{dz}{d\zeta}$$

It is to be borne in mind that only the conjugate complex value of the "complex velocity" represents the velocity vector.

A flow defined by an analytical function $F(z)$ can be mapped hydrodynamically in a number of ways. The two most frequently employed mapping methods are:

(1) $\zeta = F(z)$, in which case $\Phi(\zeta) = \zeta$, that is, the mapped space simply contains the basic flow, a horizontally directed parallel flow with the constant velocity 1. Any flow can thus be transformed to the basic flow.

(2) $\zeta = \dfrac{dF(z)}{dz} = w(z)$. This is the hodograph mapping method successfully applied by Ruden to the inlet diffuser problem.

In the following, a different mapping method is used, since, as pointed out in the introduction, an infinite series of stream functions corresponding to the different velocity ratios w_1/w_∞ must be considered for each inlet diffuser, rather than a single stream function, with which, of course, the above special mappings are invalidated. The image space is so chosen that the group of stream functions permits the simplest possible representation. A few trials confirm that the parallel strip affords the simplest solution.

Assume that the function $\zeta(z)$ maps the simply connected space situated outside the contour of the inlet diffuser single-valued and conformally on to a parallel strip bounded by the straight line $T(\zeta) = \pm\pi$. The line of symmetry of the z-space, taken parallel to the real axis, is to become the real axis of the ζ-plane, and the direction of flow is to remain the same. In the z-plane the line of symmetry connects the two infinitely remote boundary points of the space; they become with their surroundings the two tips of the parallel strip. Figure 2 represents the flow in the ζ-plane diagrammatically; the general function $\Phi(\zeta)$ which produces such a flow is given by

$$\Phi(\zeta) = c_1\zeta - c_2e^{-\zeta} \qquad (c_1 > 0,\ c_2 > 0) \tag{2}$$

To find this expression, the parallel strip is mapped by $e^{\zeta} = \zeta_1$ on the plane cut along the negative half of the real axis and the flows arising in the ζ-plane calculated. Since $\Phi'(\zeta)$ by analytical continuation beyond the strip edges obviously has the period $2\pi i$, the derivation of the transplanted stream function $\Phi_1(\zeta_1)$ must be unique. Its singularities must be looked for at 0 and ∞; at 0, corresponding to the left strip tip, a source and dipole appear combined, while at ∞, arising from the right tip, a sink occurs. As a result, the general expression of $\Phi_1(\zeta_1)$ can be immediately expressed in the form $\Phi_1(\zeta) = a \log \zeta_1 + \frac{b}{\zeta_1}$, with positive a. The outflow direction of the dipole points to the right, that is, b must be negative (real). Putting $a = c_1$, $b = -c_2$ and reverting to the ζ-plane, gives exactly the above expression.

The stagnation points are obtained by setting the derivative equal to zero. From

$$\Phi'(\zeta) = c_1 + c_2 e^{-\zeta} = 0$$

follows $e^{-\zeta} = -\frac{c_1}{c_2}$. Putting $\zeta = \xi + i\eta$ and accordingly $e^{-\zeta} = e^{-\xi - i\eta} = e^{-\xi}(\cos \eta - i \sin \eta)$ gives the conditions $e^{-\xi} \cos \eta = -\frac{c_1}{c_2}$ and $e^{-\xi} \sin \eta = 0$ for the stagnation points. Therefore, $\sin \eta = 0$, while $\cos \eta$ is negative, which in limiting ζ to the parallel strip means $\eta = \pm\pi$. Accordingly also: $e^{-\xi} = \frac{c_1}{c_2}$ and $\xi = \log \frac{c_2}{c_1}$ (natural logarithm). On putting $\zeta_o = \log \frac{c_2}{c_1} + \pi i$, that is, $e^{\zeta_o} = -\frac{c_2}{c_1}$, the expression for $\Phi(\zeta)$ can be written in the form

$$\Phi(\zeta) = c_1\left[\zeta + e^{-(\zeta - \zeta_o)}\right] \tag{3}$$

or since no additive constant is involved,

$$\Phi(\zeta) = c_1\left[(\zeta - \zeta_o) + e^{-(\zeta - \zeta_o)}\right] \tag{4}$$

The system of the streamlines is not changed by a change in the constant c_1; if ζ_o is varied - obviously $T(\zeta_o)$ must remain equal to π - the system of streamlines experiences a simple parallel displacement, as shown by the last equation.

3 - Determination of the Differential Quotient of the Function $z(\zeta)$ Which Maps the Parallel Strip on the Outside of the Inlet Diffuser

Given the function $z(\zeta)$ which maps the parallel strip on the outside of the inlet diffuser, the transplanting of the general stream function $\Phi(\zeta)$ makes the general stream function $F(z)$ available. As regards the derivation of the function $z(\zeta)$ the following may be stated:

Along the real axis, $\frac{dz}{d\zeta}$ is, on the whole, real and positive. Running through the straight line $T(\zeta) = \pi$ from left to right, $\frac{dz}{d\zeta}$ assumes at first negative (real) values in correspondence with the straight part of the contour, then, in correspondence with the curved nose contour, it assumes values which geometrically expressed belong to the lower half-plane, and lastly the values become positive (real): Letting ζ shift leftward to infinity, $\frac{dz}{d\zeta}$ must, in amount, increase beyond all limits, while at unlimited distance toward the right, $\frac{dz}{d\zeta}$ tends toward a finite (positive) limiting value. Further identification is not possible without a certain arbitrariness. The following assumption is made. Suppose quantity $\frac{dz}{d\zeta}$, when ζ passes through the upper right edge of the parallel strip, runs through a line whose center piece located in the lower halfplane is half of an ellipse symmetrical to the real axis (fig. 3).

Altogether, $\frac{dz}{d\zeta}$, considered as function in the parallel strip, gives then a conformal mapping on the shaded space represented in figure 3. For the first, the mapping of the total boundary follows from reasons of symmetry, then, the inside mapping according to the principle of the characteristics of the boundary.

Of the multitude of potential diffuser forms, one series defined by a finite number of parameters was selected. The ellipse is, for shape and position, given by three real parameters; the right-handed end point of the line, that is, $\lim_{\zeta \to +\infty} \frac{dz}{d\zeta}$, gives a fourth parameter and a fifth is ultimately afforded by the possibility of effecting any desired parallel displacement in the ζ-plane in direction of the real axis. Thus five real constants enter the general function $\frac{dz}{d\zeta}$. But, since the parallel displacements in the ζ-plane are unimportant and $\frac{dz}{d\zeta}$ itself may be multiplied by any positive factor without modifying the respective diffuser form, the factual result is a three-parametric continuum of inlet diffusers.

The analytical expression of $\frac{dz}{d\zeta}$ is obtained by mapping the parallel strip through

$$t = e^{-\zeta} \tag{5}$$

on the t-plane cut along the negative half of the real axis. The right tip takes the vicinity of the zero point, the left tip that of the infinitely remote point. The next problem of transplanting the plane thus cut on the space of figure 3 is essentially synonymous with the problem of mapping the plane fitted with a finite straight slit on the outside of an ellipse. The solution is predicated on the knowledge of the mapping attained by $z + \frac{1}{2}$. This function transforms the outside of the unit circle in the plane cut along a finite strip, and it maps the outside of the circle $|z| = R > 1$ on the outside of the ellipse; the infinitely distant point remains fixed. The outside of the circle $|z| = r < 1$ changes into an elliptically bounded space; this, however, lies in part as two-lobed space above the plane, which is to be avoided in the present instance. To pass from the slit area to the outside of an ellipse involves essentially, that is, apart from similarity mapping, the application of the inverse function of $z + \frac{1}{2}$ which gives a circular space, then, after a second similarity mapping, the transfer to the elliptical boundary according to $z + \frac{1}{2}$. Through the similarity mapping certain real constants enter in the formulas, which must be subjected to certain restrictions in order to be certain to obtain an, on the whole, single-lobed image space. The mode of calculation is as follows:

Assume that the end points lie at $-t_1$ and $-t_2$. $(t_2 > t_1 > 0)$. In that case the slit is widened out by

$$t' = t + \sqrt{(t + t_1)(t + t_2)} + \frac{t_1 + t_2}{2}$$

in a circle the center of which is the zero point of the t'-plane. This result is easily checked, because for $-t_2 < t < -t_1$ the square root is purely imaginary, and

$$\begin{aligned} |t'| &= \left|t + \frac{t_1 + t_2}{2} + \sqrt{(t + t_1)(t + t_2)}\right| \\ &= \sqrt{\left(t + \frac{t_1 + t_2}{2}\right)^2 - (t + t_1)(t + t_2)} \\ &= \sqrt{\left(\frac{t_1 + t_2}{2}\right)^2 - t_1 t_2} = \frac{t_2 - t_1}{2} \end{aligned}$$

that is, $|t'| =$ constant. The function $t'' = at' + \frac{b}{t'}$ with $a > 0$ and $b \gtreqless 0$, obtained from $z + \frac{1}{z}$ by similarity mapping results in the general ellipse whose axes coincide with the coordinate axes. To the right-hand end point of the diameter of the circle, hence to $t' = \frac{t_2 - t_1}{2}$, there must correspond a "positive" value, that is,

$$a \frac{t_2 - t_1}{2} + \frac{2b}{t_2 - t_1} > 0 \tag{6}$$

in order to prevent a two-lobed overlap of the plane at any point by the image space. Likewise, the uppermost point of the circle, $t' = i \frac{t_2 - t_1}{2}$, must take a point on the upper half plane, that is,

$$a \frac{t_2 - t_1}{2} - \frac{2b}{t_2 - t_1} > 0 \tag{7}$$

The prefix of b decides which of the coordinate axes becomes the principal axis. The desired quantity $\frac{dz}{d\zeta}$ must generally be put equal to $t'' + c$ (c real).

$$\frac{dz}{d\zeta} = a\left[t + \frac{t_1 + t_2}{2} + \sqrt{(t + t_1)(t + t_2)}\right]$$

$$+ \frac{b}{t + \frac{t_1 + t_2}{2} + \sqrt{(t + t_1)(t + t_2)}} + c$$

$$= \alpha\, t + \beta \sqrt{(t + t_1)(t + t_2)} + \gamma \tag{8}$$

with

$$\left.\begin{aligned} \alpha &= a + \frac{4b}{(t_2 - t_1)^2} \\ \gamma &= \alpha \frac{t_1 + t_2}{2} + c \\ \beta &= a - \frac{4b}{(t_2 - t_1)^2} \end{aligned}\right\} \tag{9}$$

The expression of $\frac{dz}{d\zeta}$ therefore actually contains five real constants, which must satisfy the following conditions; first, $t_2 > t_1 > 0$. The conditions (6) and (7) are equivalent to $\alpha > 0$, $\beta > 0$; $a = \frac{\alpha + \beta}{2}$ itself then becomes positive. Lastly, the ellipse must comprise the zero point even after the translation effected by the constant c. This is the case only when the values of $\frac{dz}{d\zeta}$ relating to $t = -t_1$ and $t = -t_2$ are positive and negative, respectively. From

$$-\alpha\, t_1 + \gamma > 0$$

and

$$-\alpha\ t_2 + \gamma < 0$$

follows

$$\alpha\ t_1 < \gamma < \alpha\ t_2 \tag{10}$$

The square root must be calculated positive for positive t. In the analytical continuation along the dissected t-plane, the values obtained for $t < -t_2$ are negative, while the values in the interval $-t_2 < t < -t_1$ relate to the positive and negative halfplane, depending upon whether the real axis is approached from above or below.

4.- Determination of the Mapping Function Itself by Integration. Calculation of the Contours of the Inlet Diffuser

The function $z(\zeta)$ itself is found by simple integration. From

$$\frac{dz}{d\zeta} = \alpha t + \beta \sqrt{(t + t_1)(t + t_2)} + \gamma$$

and $t = e^{-\zeta}$, that is, $\zeta = -\log t$ follows

$$\frac{dz}{dt} = \frac{dz}{d\zeta}\frac{d\zeta}{dt} = -\frac{1}{t}\frac{dz}{d\zeta} = -\alpha - \frac{\gamma}{t} - \frac{\beta}{t}\sqrt{(t + t_1)(t + t_2)}$$

and

$$z = -\alpha t - \gamma \log t - \beta \int \sqrt{(t + t_1)(t + t_2)}\ \frac{dt}{t}$$

The last integral is changed by the introduction of $\sqrt{\frac{t + t_1}{t + t_2}}$ as auxiliary variable to an integral of a rational function and computed direct. Altogether the result is

$$z = -\alpha t - \beta\sqrt{(t + t_1)(t + t_2)} - \left(\gamma + \beta\sqrt{t_1 t_2}\right)\log t$$

$$- \beta(t_1 + t_2)\log\left(\sqrt{t + t_1} + \sqrt{t + t_2}\right) + 2\beta\sqrt{t_1 t_2}\log\left(\sqrt{t_2(t + t_1)}\right. \tag{11}$$

$$\left. + \sqrt{t_1(t + t_2)}\right) + \text{constant}$$

The square roots containing t are all chosen positive for positive t-values, with which they are then unequivocally defined over the dissected t-plane in the sense of the analytical continuation; $\sqrt{t_1 t_2}$ is positive also. For the logarithmic functions which, like the square roots in the dissected t-plane, are unbranched, the principal values, that is, the values whose imaginary parts lie between $-\pi i$ and πi, are visualized as being inserted. If the additive constant is chosen real, the axis of the inlet diffuser coincides with the real axis.

On letting t run along the lower rim of the negative half of the real axis, z describes the upper contour of the desired diffuser. The intervals $(0, -t_1)$ and $(-t_2, -\infty)$ give the straight sides, the center piece $(-t_1, -t_2)$, the nose of the diffuser.

Putting $z = x + iy$,

$$x = -\alpha t - \gamma\log|t| + \text{constant},$$

$$y = \beta\sqrt{(-t_1 - t)(t_2 + t)} + \beta(t_1 + t_2)\arctan\frac{\sqrt{-t_1 - t}}{\sqrt{t_2 + t}}$$

$$- 2\beta\sqrt{t_1 t_2}\arctan\frac{\sqrt{t_2(t_1 - t)}}{\sqrt{t_1(t_2 + t)}} + \text{constant}$$

is valid along the center piece, or, if $t = -\tau$ $(t_1 \leqq \tau \leqq t_2)$ and both additive constants are taken at zero

$$x = \alpha\tau - \gamma \log \tau$$

$$y = \beta\sqrt{(\tau - t_1)(t_2 - \tau)} + \beta(t_1 + t_2) \text{ arc tan } \frac{\sqrt{\tau - t_1}}{\sqrt{t_2 - \tau}}$$

$$-2\beta\sqrt{t_1 t_2} \text{ arc tan } \frac{\sqrt{t_2(\tau - t_1)}}{\sqrt{t_1(t_2 - \tau)}} \tag{12}$$

The roots must be extracted positive, and if the argument, as here, varies between 0 and ∞, the principal branch from 0 to $\frac{\pi}{2}$ is chosen for the arc tan functions.

The formulas (12) define the upper half of the diffuser. Horizontal straight lines are drawn toward the right to infinity from the two end points of the nose given by $\tau = t_1$ and $\tau = t_2$.

$$x = \alpha t_1 - \gamma \log t_1, \; y = 0$$

and

$$x = \alpha t_2 - \gamma \log t_2, \; y = \beta(t_1 + t_2 - 2\sqrt{t_1 t_2}) \frac{\pi}{2}$$

The complete determination of the diffuser must include the inside width $2h$; from (11) follows

$$h = (\gamma + \beta \, t_1 t_2) \, \pi$$

Of the five constants which define the individual diffuser only three have any essential significance. If $\frac{dz}{d\zeta}$ is multiplied by a constant positive factor and a real constant is added to ζ, the diffuser form remains the same. But adding a real constant to ζ is reflected in t as multiplication by a positive constant. So, without causing any substantial change, the expression

$$\alpha t + \beta\sqrt{(t + t_1)(t + t_2)} + \gamma \tag{13}$$

can be multiplied by $\lambda > 0$ and t itself be replaced by $\mu t(\mu > 0)$. The expression (13) then becomes

$$\alpha\lambda\mu t + \beta\lambda\mu\sqrt{(t + \mu^{-1}t_1)(t + \mu^{-1}t_2)} + \lambda\gamma \tag{14}$$

Since α and γ are positive, because of (6) and (10), $\lambda = \frac{1}{\gamma}$ and $\mu = \frac{\gamma}{\alpha}$, and (14) reduces to

$$t + 1 + \beta'\sqrt{(t + t_1')(t + t_2')} \tag{15}$$

with

$$\beta' = \frac{\beta}{\alpha}, \quad t'_1 = \frac{\alpha}{\gamma}t_1, \quad t'_2 = \frac{\alpha}{\gamma}t_2$$

or expressed in different notation

$$\frac{dz}{d\zeta} = t + 1 + \beta\sqrt{(t + t_1)(t + t_2)} \tag{16}$$

The three quantities β, t_1, and t_2 satisfy the conditions

$$t_2 > 1 > t_1 > 0$$

and

$$\beta > 0$$

So the formulas for computing the contours read

$$x = \tau - \log \tau$$

$$y = \beta\sqrt{(\tau - t_1)(t_2 - \tau)} + \beta(t_1 + t_2) \text{ arc tan } \frac{\sqrt{\tau - t_1}}{\sqrt{t_2 - \tau}}$$

$$-2\beta\sqrt{t_1 t_2} \text{ arc tan } \frac{\sqrt{t_2(\tau - t_1)}}{\sqrt{t_1(t_2 - \tau)}} \tag{17}$$

$$h = (1 + \beta\sqrt{t_1 t_2})\pi$$

5 - Calculation and Representation of the Velocity Distribution Along the Contours

The general inlet diffuser flow is obtained by transplanting the general function $\Phi(\zeta) = c_1\zeta - c_2e^{-\zeta}$ according to (5) and (11) in the z-plane. The magnitude of the velocity along the contours is the principal point of interest. The complex velocity in the z-plane is given by

$$w(z) = \frac{dF}{dz} = \frac{d\Phi}{d\zeta} : \frac{dz}{d\zeta} \tag{18}$$

where $\frac{d\Phi}{d\zeta} = c_1 + c_2e^{-\zeta} = c_1 + c_2t$ and $\frac{dz}{d\zeta} = t + 1 + \beta\sqrt{(t + t_1)(t + t_2)}$ in the simplified method of writing. At the boundary t is negative (real). Draw a t-axis and plot the amounts of $\frac{d\Phi}{d\zeta}$ and $\frac{dz}{d\zeta}$ against it as ordinates. The graph of $\left|\frac{d\Phi}{d\zeta}\right|$ consists of two straight lines, or more exactly, pieces of straight lines, rising from the point $t = -\frac{c_1}{c_2}$ of the t-axis at the same angle toward both sides;

one straight line withdraws toward the upper left to infinity, the other terminates at height c_1 of the ordinate axis. The amount of $\frac{dz}{d\zeta}$ can be formularized as follows:

$$\left|\frac{dz}{d\zeta}\right| = \begin{cases} t + 1 + \beta\sqrt{(t + t_1)(t + t_2)} \text{ for } t \geqq t_1 \\ \sqrt{(t + 1)^2 - \beta^2(t + t_1)(t + t_2)} \text{ for } -t_1 \geqq t \geqq -t_2 \\ -t - 1 + \beta\sqrt{(t + t_1)(t + t_2)} \text{ for } t \leqq -t_2 \end{cases} \quad (19)$$

All square roots are positive. Accordingly $\left|\frac{dz}{d\zeta}\right|$ is represented by a curve that appears built up from three arcs. The two outside ones are hyperbolic arcs, while the inside one can relate to any conical section and can even be rectilinear.

Individually the following applies: Conformably to the condition $t_2 > 1 > t_1 > 0$, t_1 and t_2 are chosen fixed, so that the two end points of the central arc which are at the same time the connecting points of the outer arcs are defined. The ordinates are $1 - t_1$, and $t_2 - 1$. On examination of the entire group of curves obtained for variable β (fig. 5) it is seen that the case $\beta = 0$, regarded as limiting case, since β must be positive by assumption, results in a pair of straight lines. As β increases the ordinates along the entire line increase. The outer hyperbolic arcs deviate in their connecting points with vertical tangent and become steeper and steeper with increasing β. The inside piece cancels out hyperbolically, which is, the t-axis is the secondary axis for the first appearing hyperbolas. The curvature of the arc decreased continuously. If β reaches the value

$$\beta_0 = \frac{2\sqrt{(1 - t_1)(t_2 - 1)}}{t_2 - t_1} \quad (20)$$

the hyperbolic arc becomes the straight connection of the end points; at further increasing β the arc becomes concave downward. In general, β_0 is less than unity. The β values lying between β_0 and unity give hyperbolas for which the t-axis is principal axis. $\beta = 1$ gives a parabolic arc with the t-axis as axis of symmetry, and $\beta > 1$ results in elliptic arcs with progressively increasing steepness. The special case $\beta_0 = 1$ occurs only for $t_1 + t_2 = 2$. The end points of the middle arc are then at the same level and the convex hyperbolic arcs appearing for small β pass over the straight line corresponding to $\beta = 1$ directly in concave elliptic arcs.

The amount $|w|$ of the boundary velocity follows by (18) by division of the two functions of t; the numerator function is represented by the straight line, the denominator function is represented by the triple product of conic-section arcs.

6 - Comparison With Ruden's Investigations

Ruden (reference 1) used the hodographic method of mapping throughout his experiments; the most general space of the w-plane (w = complex velocity) taken into consideration by him is a circle the center of which lies on the real axis and which exhibits two radial incisions along the axis. The space is shown in figure 6. The slot ends w_∞ and w_i correspond to the flight speed and the terminal speed inside the diffuser, that is, the speeds relating to $t = \infty$ and $t = 0$. The values w_1 and w_2 indicate the velocity at the end points of the curved part of the diffuser wall. According to Ruden:

$$w_2 \geqq w_\infty > w_i > 0 \geqq w_1 \tag{21}$$

The analytical relation between w and t is readily indicated. The discussion is restricted to the lower half of the t-plane and of the w-circle, which are clearly and conformally referred to each other. Excluding, in the first instance, the appearance of equality signs in (21) gives the point coordination

t	0	∞	$-t_1$	$-t_2$
w	w_i	w_∞	w_1	w_2

t_1 and $t_2 > t_1$ signify any two positive values.

Mapping the lower half plane by $t' = \frac{t + t_1}{t + t_2}$ in itself in such a way that $-t_1$ and $-t_2$ change to 0 and ∞, while the value $t' = 1$ corresponds to the value $t = \infty$, 0 the formation of the square root $t'' = \sqrt{t'}$ results in a quadrant (main branch of the square root for positive t') and a linear mapping with real coefficients, which are written in the form,

$$w = \frac{at'' - b}{ct'' + d}$$

finally gives the desired w-space

Accordingly

$$w = \frac{a\sqrt{t + t_1} - b\sqrt{t + t_2}}{c\sqrt{t + t_1} + d\sqrt{t + t_2}} \tag{22}$$

The above table of values then appears as

t	0	∞	$-t_1$	$-t_2$
w	$\frac{a\sqrt{t_1} - b\sqrt{t_2}}{c\sqrt{t_1} + d\sqrt{t_2}}$	$\frac{a - b}{c + d}$	$-\frac{b}{d}$	$\frac{a}{c}$

The value $w = \infty$ corresponds to a negative t''. From it and from (21) the following conditions for the coefficients a, b, c, d are deduced:

$$\frac{a}{c} > 0, \ \frac{b}{d} > 0, \ \frac{d}{c} > 0, \ \frac{a\sqrt{t_1} - b\sqrt{t_2}}{c\sqrt{t_1} + d\sqrt{t_2}} > 0 \tag{23}$$

The first three inequalities indicate that all coefficients have the same sign; without restriction of generality they shall be positive. The last inequality signifies then

$$a > b\sqrt{\frac{t_2}{t_1}} \tag{24}$$

By making the numerator in (22) rational,

$$w = \frac{(a^2 - b^2)\ t + (a^2 t_1 - b^2 t_2)}{(ac + bd)\ t + (ad + bc)\sqrt{(t + t_1)(t + t_2)} + (act_1 + bdt_2)}$$

Quantity $a^2 - b^2$ is positive by reason of (24). Since the coefficients a, b, c, and d may be multiplied by any positive constant, without modifying $w(t)$, $a^2 - b^2 = 1$ (25) may be assumed. Therefore:

$$w = \frac{t + t_o}{At + B\sqrt{(t + t_1)(t + t_2)} + C} \tag{26}$$

with

$$\left.\begin{aligned} t_o &= a^2 t_1 - b^2 t_2 \\ A &= ac + bd \\ B &= ad + bc \\ C &= act_1 + bdt_2 \end{aligned}\right\} \tag{27}$$

A, B, C, and t_o are positive. On comparing the expression for w in (26) with

$$w = \frac{c_2 t + c_1}{\alpha t + \beta\sqrt{(t + t_1)(t + t_2)} + \gamma}$$

which can also be written in the form

$$w = c_2 \frac{t + t_o}{\alpha t + \beta\sqrt{(t + t_1)(t + t_2)} + \gamma} \tag{28}$$

where the denominator is used again in full generality, it is seen that all the forms considered by Ruden are included. The only conditions which the present coefficients, aside from that of being positive, must satisfy are: $t_2 > t_1 > 0$ and (10); $t_2 > t_1 > 0$ is also satisfied for (26) and from (27) follows immediately the inequality corresponding to (10)

$$At_1 < C < At_2$$

Two more facts stand out:

(1) The numerator in (26) is defined when the denominator is known; this is proved by (25) and (27). But the zero place of the denominator in (28) remains arbitrary for given numerator (<0). (The factor c_2 is naturally unessential.) The limitation in (26) is associated with the nature of the hodographic method, which momentarily comprises only a specific operating condition and does not detract from the generality as far as the diffuser forms are concerned.

(2) It is readily seen that for specified values of t_1 and t_2 the formulas (25) and (27) do not permit the ratios A : B : C to vary in the same manner as the ratios $\alpha : \beta : \gamma$. Thus at $t_1 = 1$ and $t_2 = 2$ the proportional equation $A : B : C = 1 : 1 : \frac{3}{2}$ can certainly not be made to agree with (27), while being able to put $\alpha = \beta = 1$ and $\gamma = \frac{3}{2}$. To this extent the formula is also more general as regards

diffuser forms than Ruden's formula (in reference 1) although the number of constants is not greater. Ruden's study of diffusers showing a constant speed along the nose contour in normal operating condition requires, according to the present theory, linearity of the curve of $\left|\frac{dz}{d\zeta}\right|$ representing the central piece. The normal operating conditions further require that the zero place of the numerator function of (28) lie, geometrically speaking, in the straight extension of the central piece. The diffusers characterized in the foregoing by $\beta = \beta_o$ correspond accordingly to these special inlet diffusers. The degree of generality of these investigations surpassing Ruden (reference 1) in two ways, is especially clear. On the one hand, the numerator function in the expression of w need not disappear exactly at the t-place lying in the extension of the center piece (more general operating condition), on the other, this extension need not as in Ruden's report, intersect the t-axis at all in a point with negative abscissa (more general diffuser forms).

II - THEORY OF OPTIMUM DIFFUSERS

1 - The Characteristic Quantities w_1, w_∞, w_{max} and $w_*\delta$

Earlier in the present report the complex velocity of the general inlet diffuser flow was defined and the amount of the velocity along the diffuser contour analyzed. This amount is hereinafter designated by w; the complex velocity previously denoted by w does no longer appear. With $s(t)$ as the magnitude of the quantity $\frac{dz}{d\zeta}$ along the boundary

$$s(t) = \begin{cases} t + 1 + \beta\sqrt{(t + t_1)(t + t_2)} & \text{for} \quad t \geqq -t_1 \\ \sqrt{(t + 1)^2 - \beta^2(t + t_1)(t + t_2)} & \text{for} \quad -t_1 \geqq t \geqq -t_2 \\ -t - 1 + \beta\sqrt{(t + t_1)(t + t_2)} & \text{for} \quad t \leqq -t_2 \end{cases} \tag{29}$$

It is borne in mind that t assumes only negative values. So far w as function of t

$$w(t) = \frac{|c_1 + c_2 t|}{s(t)} = c_2\frac{|t + t_o|}{s(t)} \tag{30}$$

In the subsequent study only velocity ratios will be the controlling factors. Without restriction of generality c_2 can be put as $c_2 = 1$. To the values $t = 0$ and $t = -\infty$ correspond the velocities w_1 and w_∞; they are readily obtained when t_o, t_1, t_2, and β are known. On the other hand, it is quite difficult to set up a general expression for maximum increase of speed w_{max} relating to the given parameter values. In each specific case w_{max} is also easily calculated. To establish the abscissa value for which w attains its maximum, consider a pair of straight lines (fig. 8), radiating from point $(-t_o, 0)$ and symmetrical to the vertical $t = -t_o$. Assume that this set of straight lines, starting from a very flat slope with respect to the t-axis, becomes progressively steeper until it meets the curve C representing $s(t)$ for the first time. The intersection point gives the desired abscissa value. Actually, the pair of straight lines can be regarded as geometric representation of the numerator of w in the particular limiting position, since no proportionality factor is involved. In that case $w \leqq 1$ and the equal sign is reached exactly in that particular point. Naturally, w_{max} can occur at several points. The difficulty of a general determination of w_{max} arises from the fact that different cases have to be distinguished, depending upon whether the maximum point lies within the central arc of C or at a point of discontinuity. The general examination is restricted to the determination in place of w_{max} to the higher of the two velocities which correspond to the two points of discontinuity, indicated as w_* and noting that in many cases $w_* = w_{max}$. For the equation $w_* = w_{max}$ certainly applies when C has a straight or even slightly convex central piece, and indeed, it is then fulfilled for all values of t_o. It also applies for slightly concave central piece in certain conditions. The subsequently discussed optimum diffuser forms show this characteristic practically generally, which physically implies that the maximum increase of speed occurs at one of the two end points of the diffuser nose. At any rate the difference of w_{max} and w_* in the diffuser types treated in section IV is significant. The value of $t = -t_1$ or $t = -t_2$ corresponding to the higher speed is readily apparent. At fixed $s(t)$ and variable t_o a reversal takes place when t_o passes through unity, or when the apex of the two straight lines falls in the straight extension of the line connecting the points of discontinuity of $s(t)$. With $-t_3$ signifying the abscissa of the point of intersection of this extension with the t-axis

$$t_3 = \frac{2t_1t_2 - t_1 - t_2}{t_1 + t_2 - 2} \qquad (31)$$

After examination of the different positions successively the result is as follows:

(a) $t_3 \leqq 0$

(α) $t_o \leqq 1 \quad w_1 : w_\infty : w_* = \dfrac{t_o}{1 + \beta\sqrt{t_1 t_2}} : \dfrac{1}{1 + \beta} : \dfrac{t_2 - t_o}{t_2 - 1}$

(β) $t_o \geqq 1 \quad w_1 : w_\infty : w_* = \dfrac{t_o}{1 + \beta\sqrt{t_1 t_2}} : \dfrac{1}{1 + \beta} : \dfrac{t_o - t_1}{1 - t_1}$

(b) $0 \leqq t_3 < 1$ (32)

(α) $t_o \leqq t_3 \quad w_1 : w_\infty : w_* = \dfrac{t_o}{1 + \beta\sqrt{t_1 t_2}} : \dfrac{1}{1 + \beta} : \dfrac{t_1 - t_o}{1 - t_1}$

(β) $t_3 \leqq t_o \leqq 1 \quad w_1 : w_\infty : w_* = \dfrac{t_o}{1 + \beta\sqrt{t_1 t_2}} : \dfrac{1}{1 + \beta} : \dfrac{t_2 - t_o}{t_2 - 1}$

(γ) $t_o \geqq 1 \quad w_1 : w_\infty : w_* = \dfrac{t_o}{1 + \beta\sqrt{t_1 t_2}} : \dfrac{1}{1 + \beta} : \dfrac{t_o - t_1}{1 - t_1}$

(c) $t_3 > 1$

(α) $t_o \leqq 1 \quad w_1 : w_\infty : w_* = \dfrac{t_o}{1 + \beta\sqrt{t_1 t_2}} : \dfrac{1}{1 + \beta} : \dfrac{t_2 - t_o}{t_2 - 1}$

(β) $1 \leqq t_o \leqq t_3 \quad w_1 : w_\infty : w_* = \dfrac{t_o}{1 + \beta\sqrt{t_1 t_2}} : \dfrac{1}{1 + \beta} : \dfrac{t_o - t_1}{1 - t_1}$

(32)

(γ) $t_o \geqq t_3 \quad w_1 : w_\infty : w_* = \dfrac{t_o}{1 + \beta\sqrt{t_1 t_2}} : \dfrac{1}{1 + \beta} : \dfrac{t_o - t_2}{t_2 - 1}$

The case $t_3 = 1$ does not occur; $t_3 = \infty$, that is $t_1 + t_2 = 2$ can be incorporated under (a) or (c) (α, β).

The ratio of the speeds w_1, w_∞, and w_* permits an appraisal of the mode of operation of the inlet diffuser. In addition the wall thickness d, measured, say in ratio of 1/2 inside width, is important. By (11)

$$\delta = \frac{d}{h} = \frac{\left(\beta \frac{t_1 + t_2}{2} - \sqrt{t_1 t_2}\right)}{1 + \beta\sqrt{t_1 t_2}} \tag{33}$$

The question posed in the introduction concerning the best possible design of diffuser contour can now be formulated as follows:

The parameters t_1, t_2, and β shall be so defined that

(1) The quantity δ becomes as small as possible

(2) At variation of $w_1 : w_\infty$ within a specific range, that is, at variation of to within a corresponding interval, the quantity w_{max} (or w_*) is the smallest possible with respect to the higher of the two velocities w_∞ and w_1.

2 - The Question of Most Favorable Choice of Parameter

A mathematically precise treatment of the optimum problem presents great difficulties even with w_{max} instead of w_*. There are two reasons for these complications. First, it is, of course, easy, for a fixed ratio $w_1 : w_\infty$, to make the problem a precise minimum problem. Either assume fixed δ and define t_1, t_2, β, and t_o in such a way that (33) is complied with and $\frac{w_*}{w_\infty}$ becomes a minimum, or else specify $\frac{w_*}{w_\infty}$ and attempt to reduce δ to a minimum. But at variable $w_1 : w_\infty$ on the other hand, $\frac{w_*}{w_\infty}$ is naturally variable also, and a precise formulation of the minimum problem is not possible without option. Second, the complicated form of the function $s(t)$ pieced together from several analytical functions is disturbingly noticeable in the calculations and the attempt to restore the organic character of a uniform function introduces new difficulties. For this reason it was decided to set up rules based upon the geometrical view according to which the parameter values are chosen to suit the purpose. Everything else is left to the special numerical calculation.

(a) The case of constant velocity ratio $w_1 : w_\infty$ is analyzed first. The numerator function of the expression for w, represented geometrically by a pair of straight lines (g_1', g_2'), is visualized as fixed. If a certain increase of velocity is admitted, the curve C representing the denominator function $s(t)$ must be situated in a part of the (t, s) plane which is downwardly bounded by a certain pair of straight lines (g_1, g_2). This pair has the same vertex point as (g_1', g_2') and has, like (g_1', g_2'), a vertical line of symmetry. Even (g_1, g_2) can be regarded as geometrical pattern of the numerator function, since constant factors are not involved. The best diffuser is obtained when the infinite angular space available for C is utilized to the best advantage, that is, the curve is as close as possible to the boundary. There are two possibilities. The first consists in choosing the central piece of C straight and placing it immediately on g_1 or g_2. This way a two-parametric continuum is separated from the three-parametric diffuser continuum defined by t_1, t_2, and β; β_0 is obtained (20) through t_1 and t_2. This two-parametric continuum decomposes in two partial groups; one, characterized by g_2 as carrier of the straight part of C, is the group analyzed by Ruden particularly. The two parameters t_1 and t_2 permit, as will be seen, the realization of any velocity ratio $w_1 : w_\infty : w_{max}$. For Ruden's group, $w_1 < w_\infty$.

The second possibility consists in placing a point of discontuity of C on both g_1 and g_2 and choosing β very small so that the central piece of C clings very closely to g_1 and g_2. The limiting case $\beta = 0$ leads direct to the pair of straight lines itself; the corresponding diffuser consists of two infinitely thin straight walls and represents the optimum solution for $w_1 = w_\infty$.

(b) Suppose the ratio $\frac{w_1}{w_\infty}$ is variable. The case of unlimited variation serves as basis, but w_1 and w_∞ themselves are visualized as varying only between zero and a finite limit. In this event the fixed numerator function previously represented by (g_1, g_2) is replaced by a two-parametric continuum of numerator functions. What part of the plane do the corresponding pairs of straight lines cover?

The intersection points with the ordinate axis cover a finite interval starting from the zero point, while the vertex points cover the entire negative half of the t-axis. To vertex points of very great distance correspond very flat pairs of straight lines, that is, very small values of w_∞. The measure of rise varies altogether between zero and a finite value as exemplified in figure 9. The space filled by the pairs of straight lines is itself bounded by a pair of straight lines which again is denoted by (g_1, g_2).

3 - Theory of Best Diffusers for Fixed $w_1 : w_\infty$

The diffusers for which the curve C exhibits a straight central piece are characterized by the condition

$$\beta = \beta_o = \frac{2\sqrt{(1 - t_1)(t_2 - 1)}}{t_2 - t_1} \tag{34}$$

The line connecting the points of discontinuity meets the abscissa axis in point $-t_3$, where t_3 shall now be positive. Putting $t_o = t_3$ (31)) gives the normal operating condition of the diffuser, for the point $(-t_3, 0)$ corresponds to the vertex point of (g_1, g_2). Since $w_{max} = w_*$, by (32)

$$w_1 : w_\infty : w_{max} = \frac{2t_1t_2 - t_1 - t_2}{(t_1 + t_2 - 2)(1 + \beta\sqrt{t_1t_2})} : \frac{1}{1 + \beta} : \frac{t_2 - t_1}{t_1 + t_2 - 2} \text{ for } t_3 < 1$$

$$w_1 : w_\infty : w_{max} = \frac{t_1 + t_2 - 2t_1t_2}{(2 - t_1 - t_2)(1 + \beta\sqrt{t_1t_2})} : \frac{1}{1 + \beta} : \frac{t_2 - t_1}{2 - t_1 - t_2} \text{ for } t_3 > 1 \tag{35}$$

for β the value (34) is inserted.

Equations (34) and (35) are now used to compute t_1, t_2, and β for prescribed ratios $w_1 : w_\infty : w_{max}$. From

$$\beta^2 = \frac{4(t_1 + t_2 - 1 - t_1t_2)}{(t_2 - t_1)^2}$$

follows

$$1 - \beta^2 = \frac{(t_1 + t_2 - 2)^2}{(t_2 - t_1)^2} = (1 + \beta)^2 \frac{w_\infty^2}{w_{max}^2}$$

hence

$$\frac{w_\infty^2}{w_{max}^2} = \frac{1 - \beta^2}{(1 + \beta)^2} = \frac{1 - \beta}{1 + \beta}$$

or

$$\beta = \frac{w_{max}^2 - w_\infty^2}{w_{max}^2 + w_\infty^2} \tag{36}$$

The application of this value gives

$$\frac{(t_1 + t_2 - 2)^2}{(t_2 - t_1)^2} = \frac{4w_{max}^2 w_\infty^2}{(w_{max}^2 + w_\infty^2)^2}$$

or

$$\frac{t_1 + t_2 - 2}{t_2 - t_1} = \pm \frac{2w_{max} w_\infty}{w_{max}^2 + w_\infty^2} \tag{37}$$

Let t_3 be less than 1, that is, $t_1 + t_2 - 2$ be greater than 0. In this case (37) carries the plus sign; and (37) is written in the form

$$\frac{(t_2 - 1) - (1 - t_1)}{(t_2 - 1) + (1 - t_1)} = \frac{2w_{max} w_\infty}{w_{max}^2 + w_\infty^2}$$

hence

$$\frac{t_2 - 1}{1 - t_1} = \frac{(w_{max} + w_\infty)^2}{(w_{max} - w_\infty)^2}$$

According to this relation put

$$1 - t_1 = \lambda(w_{max} - w_\infty)^2; \quad t_2 - 1 = \lambda(w_{max} + w_\infty)^2$$

or

$$t_1 = 1 - \lambda(w_{max} - w_\infty)^2; \quad t_2 = 1 + \lambda(w_{max} + w_\infty)^2$$

Quantity w_1 must be used for determining λ.

$$t_3 = \frac{2t_1t_2 - t_1 - t_2}{t_1 + t_2 - 2} = 1 - \frac{\lambda}{2}\frac{(w_{max}^2 - w_\infty^2)^2}{w_{max}\, w_\infty}$$

and after a simple calculation

$$\frac{w_1}{w_{max}} = \frac{2w_{max}\, w_\infty - \lambda(w_{max}^2 - w_\infty^2)^2}{w_{max}^2 + w_\infty^2 + (w_{max}^2 - w_\infty^2)\sqrt{\left[1 + \lambda(w_{max} + w_\infty)^2\right]\left[1 - \lambda(w_{max} - w_\infty)^2\right]}}$$

This equation contains only λ as unknown. The removal of denominator and square root leaves an ordinary quadratic equation for λ. Its two roots are

$$\lambda_1 = \frac{2w_{max}(w_{max}^2 - w_1 w_\infty)(w_\infty - w_1)}{(w_{max}^2 - w_\infty^2)^2\,(w_{max}^2 + w_1^2)}$$

and

$$\lambda_2 = \frac{2w_{max}\,w_\infty}{\left(w_{max}^2 - w_\infty^2\right)^2}$$

The second solution proves useless or at least dispensable. It leads to $t_o = 0$, a result that applies only to $w_i = 0$. But for $w_i = 0$, λ_1 gives a value that exactly agrees with λ_2. Therefore put $\lambda = \lambda_1$, so that

$$t_1 = \frac{w_{max}^2 + w_\infty^2}{w_{max}^2 + w_i^2}\,\frac{(w_{max} + w_i)^2}{(w_{max} + w_\infty)^2} \tag{38}$$

and

$$t_2 = \frac{w_{max}^2 + w_\infty^2}{w_{max}^2 + w_i^2}\,\frac{(w_{max} - w_i)^2}{(w_{max} - w_\infty)^2}$$

For the present the calculation gives t_1 and t_2 in the form

$$t_1 = 1 - \frac{2w_{max}\left(w_{max}^2 - w_i w_\infty\right)(w_\infty - w_i)}{\left(w_{max}^2 + w_i^2\right)(w_{max} + w_\infty)^2}$$

$$t_2 = 1 + \frac{2w_{max}\left(w_{max}^2 - w_i w_\infty\right)(w_\infty - w_i)}{\left(w_{max}^2 + w_i^2\right)(w_{max} - w_\infty)^2} \tag{39}$$

These expressions indicate that $t_2 > 1 > t_1$ only when $w_i < w_\infty$; therefore (38) applies only to this case. This limitation is linked with the previously made limiting assumption $t_3 < 1$. As is immediately apparent from (39), $t_1 + t_2 - 2$ is actually > 0, that is, $t_3 < 1$ for $w_i < w_\infty$. For $t_1 + t_2 - 2 < 0$ the minus sign is carried in (37). Then

$$\frac{t_2 - 1}{1 - t_1} = \frac{(w_{max} - w_\infty)^2}{(w_{max} + w_\infty)^2}$$

and a similar equation as above gives

$$t_1 = \frac{{w_{max}}^2 + {w_\infty}^2}{{w_{max}}^2 + {w_i}^2} \frac{(w_{max} - w_i)^2}{(w_{max} - w_\infty)^2}$$

$$t_2 = \frac{{w_{max}}^2 + {w_\infty}^2}{{w_{max}}^2 + {w_i}^2} \frac{(w_{max} + w_i)^2}{(w_{max} + w_\infty)^2} \tag{40}$$

Thus compared to (38) only the expressions for t_1 and t_2 are exchanged. It is easily checked that $t_2 > 1 > t_1$ is exactly fulfilled for $w_i > w_\infty$, and that for $w_i > w_\infty$ actually $t_1 + t_2 - 2 < 0$, as it must be.

With it the problem of defining β, t_1, and t_2 for prescribed ratios $w_i : w_\infty : w_{max}$ is completely solved. For $w_i < w_\infty$ equations (36) and (38), for $w_i > w_\infty$, (36) and (40) must be applied. The case of $w_i = w_\infty$ is technically of no interest and mathematically trivial.

In conclusion, the quantity δ is computed.

$$\sqrt{t_1 t_2} = \frac{{w_{max}}^2 + {w_\infty}^2}{{w_{max}}^2 + {w_i}^2} \frac{{w_{max}}^2 - {w_i}^2}{{w_{max}}^2 - {w_\infty}^2}$$

$$1 + \beta\sqrt{t_1 t_2} = \frac{2w_{max}^2}{w_{max}^2 + w_i^2}$$

$$\frac{t_1 + t_2}{2} = \frac{w_{max}^2 + w_\infty^2}{w_{max}^2 + w_i^2}\,\frac{w_{max}^4 + w_{max}^2(w_\infty^2 + w_i^2 - 4w_i w_\infty) + w_i^2 w_\infty^2}{(w_{max}^2 - w_\infty^2)^2}$$

$$\beta\,\frac{t_1 + t_2}{2} = \frac{w_{max}^4 + w_{max}^2(w_\infty^2 + w_i^2 - 4w_i w_\infty) + w_i^2 w_\infty^2}{(w_{max}^2 + w_i^2)(w_{max}^2 - w_\infty^2)}$$

whence after an easy calculation

$$\delta = \frac{(w_\infty - w_i)^2}{w_{max}^2 - w_\infty^2} \tag{41}$$

This important formula of Ruden's theory is obtainable without difficulty; it is valid for $w_i \lesseqgtr w_\infty$.

Another interesting and practically important result is the following: the quantities denoted by w_i, w_∞, and w_{max} correspond to the normal operating condition and are now written with capital letters. When any desired operating conditions for a fixed diffuser are to be studied, the equation system (32) is used with general t_o instead of (35); w_* can then also be replaced by w_{max}. By limitation to $w_i < w_\infty$, and taking (36) and (38) into account where capital letters are used at the right-hand side,

$$w_i:w_\infty:w_{max} = t_o\frac{W_{max}^2 + W_i^2}{2W_{max}^2}:\frac{W_{max}^2 + W_\infty^2}{2W_{max}^2}:\begin{cases} \dfrac{t_1 - t_o}{1 - t_1} & \text{for } t_o \leqq t_3 \\[2ex] \dfrac{t_2 - t_o}{t_2 - 1} & \text{for } t_3 \leqq t_o \leqq 1 \\[2ex] \dfrac{t_o - t_1}{1 - t_1} & \text{for } t_o \geqq 1 \end{cases}$$

whence

$$t_o = \frac{w_1}{w_\infty} \frac{W_{max}^2 + W_\infty^2}{W_{max}^2 + W_1^2}$$

When the ratio $\frac{w_1}{w_\infty}$ gradually increases from 0, t_o increases correspondingly. The ratio $\frac{w_{max}}{w_\infty}$ decreases continuously so long as t_o remains less than unity; then it rises again for $t_o > 1$. Thus the smallest increase of speed occurs at $t_o = 1$, that is, for

$$\frac{w_1}{w_\infty} = \frac{W_{max}^2 + W_1^2}{W_{max}^2 + W_\infty^2}$$

In figure 15 the velocities at the end points of the nose of a specific Ruden inlet diffuser $\left(\frac{W_1}{W_\infty} = 0.4; \frac{W_{max}}{W_\infty} = 1.2\right)$ are represented divided by the flight speed, as function of $\frac{w_1}{w_\infty}$; the higher of the two speeds is w_{max}. The diffuser shows a distinctly demarcated range of favorable efficiency. This range is given by

$$\frac{W_1'}{W_\infty} \leqq \frac{w_1}{w_\infty} \leqq \frac{W_{max}^2 + W_1^2}{W_{max}^2 + W_\infty^2}$$

If δ and $\frac{W_{max}}{W_\infty}$ are given, $\frac{W_1}{W_\infty}$ can be computed by (41). Figure 14 in Ruden's report (reference 1) represents this relation graphically by the curves $\frac{W_1}{W_\infty}$ = constant in a $\left(\frac{W_{max}}{W_\infty}, \delta\right)$ plane. This figure is now complemented by the addition of the curves $\frac{W_{max}^2 + W_1^2}{W_{max}^2 + W_\infty^2}$ = constant; $\left(\frac{w_1}{w_\infty}\right)_n$ = constant replaces $\frac{W_1}{W_\infty}$ = constant, $\left(\frac{w_1}{w_\infty}\right)_o$ = constant replaces $\dfrac{W_{max}^2 + W_1^2}{W_{max}^2 + W_\infty^2}$ = constant in order to indicate that the upper and lower limit of the favorable efficiency range are included. Figure 10 makes it possible to define the limits of the favorable efficiency range for prescribed values of δ and $\dfrac{W_{max}}{W_\infty}$.

The second diffuser type for fixed $w_1 : w_\infty$ is mathematically very simple. The vertex of the pair of straight lines is placed toward (-1,0), while the straight lines themselves rise at 45°; t_o is put equal to unity; w_* coincides with w_{max}. By (32)

$$w_1 : w_\infty : w_{max} = \frac{1}{1 + \beta\sqrt{t_1 t_2}} : \frac{1}{1 + \beta} : 1$$

hence

$$\beta = \frac{w_{max} - w_\infty}{w_\infty} \tag{42}$$

$$\sqrt{t_1 t_2} = \frac{w_{max} - w_1}{w_{max} - w_\infty}\,\frac{w_\infty}{w_1} \tag{43}$$

For δ

$$\delta = \frac{w_{max} - w_\infty}{w_\infty}\,\frac{w_1}{w_{max}}\,\frac{t_1 + t_2}{2} - \frac{w_{max} - w_1}{w_{max}}$$

To keep δ at a minimum, quantity $\dfrac{t_1 + t_2}{2}$ must be reduced to a minimum. The geometric mean of t_1 and t_2, $\sqrt{t_1 t_2}$ is defined by (43); the arithmetic mean $\frac{1}{2}(t_1 + t_2)$ becomes a minimum when t_1 and t_2 are made to come together as closely as possible. As $t_2 > 1 > t_1$ is to be valid, the optimum is only approximately attainable. For $w_1 < w_\infty$, $\sqrt{t_1 t_2} > 1$, and the optimum is represented by

$$t_1 = 1,\ t_2 = t_1 t_2 = \frac{(w_{max} - w_1)^2}{(w_{max} - w_\infty)^2}\frac{w_\infty{}^2}{w_1{}^2}$$

For $w_1 > w_\infty$ these values must be exchanged for t_1 and t_2. In any case

$$\frac{t_1 + t_2}{2} = \frac{w_{max}{}^2(w_\infty{}^2 + w_1{}^2) - 2w_{max}w_1w_\infty(w_1 + w_\infty) + 2w_1{}^2\,w_\infty{}^2}{2w_1{}^2(w_{max} - w_\infty)^2}$$

and

$$\delta = \frac{w_{max}(w_\infty - w_1)^2}{2w_1\,w_\infty(w_{max} - w_\infty)} \tag{44}$$

This value is now indicated by δ_* and then compared with the values given by (41):

$$\delta_* = \delta\,\frac{w_{max}(w_{max} + w_\infty)}{2w_1\,w_\infty}$$

The quotient $\frac{\delta_*}{\delta}$ is in any case greater than unity. The second diffuser type is, therefore, inferior to the first. For very small and very great $w_1 : w_\infty$, δ_* must become substantially greater than δ. Because for $w_1 < w_\infty$:

$$\frac{w_{max}(w_{max} + w_\infty)}{2w_1\,w_\infty} > \frac{w_\infty 2w_\infty}{2w_1\,w_\infty} = \frac{w_\infty}{w_1} \tag{45}$$

and for $w_1 > w_\infty$:

$$\frac{w_{max}(w_{max} + w_\infty)}{2w_1\,w_\infty} > \frac{w_1(w_1 + w_\infty)}{2w_1\,w_\infty} = \frac{1}{2}\left(\frac{w_1}{w_\infty} + 1\right) \tag{46}$$

so that $\frac{w_1}{w_\infty}$ constitutes a measure for the minimum increase which δ_* experiences with respect to δ. In (45) and (46), w_{max} was replaced by the greater of the speeds w_1 and w_∞. Consequently, the estimates are much better as the speed increases are smaller. But small speed increases correspond to small parameter values β.

4.- Theory of Optimum Diffusers for Any Variable Ratio $w_1 : w_\infty$

In this instance three types of diffusers are involved. For the first two, C has a straight central piece that lies on g_2 or g_1; for the third type the central piece of C is curved and touches g_1 and g_2. The straight line g_1 runs parallel to the t-axis, while g_2 passes through the origin of the coordinates.

The first type was theoretically discussed; it belongs to the Ruden group. Now, however, the reference point t_o must be chosen general, and at the same time the relation $t_3 = 0$, that is

$$2t_1t_2 - t_1 - t_2 = 0 \tag{47}$$

must be observed. By (34)

$$\beta^2 = \frac{4(t_1 + t_2 - 1 - t_1t_2)}{(t_2 - t_1)^2};$$

the insertion of

$$t_2 = \frac{t_1}{2t_1 - 1}$$

according to (47) gives

$$\beta^2 = \frac{2t_1 - 1}{t_1^{\,2}}$$

or differently expressed

$$\beta^2 = \frac{1}{t_1 t_2}$$

Accordingly δ is computed as follows:

$$\delta = \frac{\beta}{1 + \beta\sqrt{t_1 t_2}}\left(\frac{t_1 + t_2}{2} - \sqrt{t_1 t_2}\right) = \frac{1}{2}\left(\sqrt{t_1 t_2} - 1\right)$$

Since $w_{max} = w_*$ the velocity ratio follows as

$$w_1 : w_\infty : w_{max} = \frac{t_o}{2} : \frac{1}{1 + \sqrt{\frac{1}{t_1 t_2}}} : \begin{cases} \dfrac{t_2 - t_o}{t_2 - 1} \text{ for } t_o \leqq 1 \\ \dfrac{t_o - t_1}{1 - t_1} \text{ for } t_o \geqq 1 \end{cases}$$

Being chiefly interested in the case of very high values of $w_1 : w_\infty$, t_o is accordingly put as $t_o \longrightarrow \infty$, so that

$$w_{max} : w_1 = \frac{2}{1 - t_1}$$

It is then readily apparent that t_1 must be greater than $\frac{1}{2}$. Therefore

$$\frac{w_{max}}{w_1} > \frac{2}{1 - \frac{1}{2}} = 4$$

However, since speed increases of this order of magnitude are inadmissable in practice, the first type is unsuitable for the present purpose.

For the second type, $\beta = 1$ and $t_1 + t_2 = 2$. Again $w_{max} = w_*$, hence

$$w_i : w_\infty : w_{max} = \frac{t_o}{1 + \sqrt{t_1 t_2}} : \frac{1}{2} : \begin{cases} \dfrac{t_2 - t_o}{t_2 - 1} \text{ for } t_o \leqq 1 \\ \dfrac{t_o - t_1}{1 - t_1} \text{ for } t_o \geqq 1 \end{cases}$$

For very high t_o values

$$w_{max} : w_i = \frac{1 + \sqrt{t_1 t_2}}{1 - t_1} = \frac{1 + \sqrt{1 - a^2}}{a}$$

with $t_1 = 1 - a$, $t_2 = 1 + a$. By proper selection of a the last ratio can be made to approach unity as closely as desired, so that in this respect the conditions are much more propitious than in the former case. On the other hand, the conditions for medium speed ratios are still too unfavorable. For from

$$t_o = \frac{1 + \sqrt{t_1 t_2}}{2} \frac{w_i}{w_\infty} = \frac{1 + \sqrt{1 - a^2}}{2} \frac{w_i}{w_\infty}$$

when assuming $\frac{w_i}{w_\infty} < 1$ and hence $t_o < 1$, follows

$$\begin{aligned} w_{max} &= 2 \frac{t_2 - t_o}{t_2 - 1} w_\infty = 2 \frac{1 + a - t_o}{a} w_\infty \\ &= 2w_\infty + \frac{2}{a}\left(w_\infty - \frac{1 + \sqrt{1 - a^2}}{2} w_i\right) > 2w_\infty \end{aligned}$$

a result useless in practice.

The third type is characterized by the fact that C touches g_1 and g_2. The points of contact have the abscissas $-t_1$ and $-t_2$. The equation of the central arc of C is by (29):

$$s^2 = (t + 1)^2 - \beta^2(t + t_1)(t + t_2)$$

which, after differentiation and putting $t = -t_1$, gives

$$2s \left.\frac{ds}{dt}\right|_{t=-t_1} = 2(1 - t_1) - \beta^2(t_2 - t_1)$$

By assumption this expression must disappear, whence

$$\beta^2 = 2 \frac{1 - t_1}{t_2 - t_1} \tag{48}$$

This value, introduced in the expression of s^2 and differentiated for $t = -t_2$, gives

$$2s \left.\frac{ds}{dt}\right|_{t=-t_2} = 2(1 - t_2) + 2 \frac{1 - t_1}{t_2 - t_1} (t_2 - t_1) = 2(2 - t_1 - t_2)$$

or, since $s(-t_2) = t_2 - 1$:

$$\left.\frac{ds}{dt}\right|_{t=-t_2} = \frac{2 - t_1 - t_2}{t_2 - 1}$$

In order that C touch g_2 this value must be equal to $-\frac{t_2 - 1}{t_2}$, that is,

$$(2 - t_1 - t_2)\, t_2 + (t_2 - 1)^2 = 0$$

or

$$t_1 t_2 = 1 \tag{49}$$

By (48) and (49)

$$\beta^2 = \frac{2t_1}{1 + t_1}$$

or

$$t_1 = \frac{\beta^2}{2 - \beta^2}$$

Quantity δ is expressed by β in the following manner:

$$\delta = \frac{\beta}{2(1 + \beta)}\left(\frac{\beta^2}{2 - \beta^2} + \frac{2 - \beta^2}{\beta^2} - 2\right) = \frac{2(1 + \beta)(1 - \beta)^2}{\beta(2 - \beta^2)}$$

The connection between β and δ is numerically represented in the table as follows:

β	1.0	0.9	0.8	0.7	0.6	0.5	0.4
δ	0	0.03	0.13	0.29	0.50	0.86	1.39

To keep δ within tolerable limits, β certainly must not fall below 0.5. Formulas (32) then give ($w_* = w_{max}$):

$$w_1 : w_\infty : w_{max} = \frac{t_o}{1 + \beta} : \frac{1}{1 + \beta} : \begin{cases} \dfrac{2 - \beta^2 - \beta^2 t_o}{2(1 - \beta^2)} & \text{for } t_o \leqq 1 \\[2ex] \dfrac{(2 - \beta^2)\, t_o - \beta^2}{2(1 - \beta^2)} & \text{for } t_o \geqq 1 \end{cases}$$

that is

$$t_o = \frac{w_1}{w_\infty} \tag{51}$$

and

$$\left.\begin{aligned} w_{max} &= \frac{2 - \beta^2}{2(1 - \beta)} w_\infty - \frac{\beta^2}{2(1 - \beta)} w_i \text{ for } w_i \leqq w_\infty \\ w_{max} &= \frac{2 - \beta^2}{2(1 - \beta)} w_i - \frac{\beta^2}{2(1 - \beta)} w_\infty \text{ for } w_i \geqq w_\infty \end{aligned}\right\} \tag{52}$$

for $\beta = \frac{1}{2}$, for example,

$$w_{max} = \begin{cases} 1.75w_\infty - 0.25w_i(w_i \leqq w_\infty) \\ 1.75w_i - 0.25w_\infty(w_i \geqq w_\infty) \end{cases}$$

The conditions are seen to be much more favorable than in the other two cases. Nevertheless it is desirable to reduce the speed increases still farther without increasing δ. This is accomplished, geometrically speaking, by placing the central arc of C a little lower. Analytically, this operation signifies a decrease of β for fixed values of t_1 and t_2. The corresponding wall thickness $\delta = \frac{\beta}{1 + \beta}\left(\frac{t_1 + t_2}{2} - 1\right)$ decreases with decreasing β, and, (51) being valid, the velocity w_* follows as

$$\left.\begin{aligned} w_* &= \frac{1 + \beta}{1 - t_1} (w_\infty - t_1 w_i) \text{ for } w_i \leqq w_\infty \\ w_* &= \frac{1 + \beta}{1 - t_1} (w_i - t_1 w_\infty) \text{ for } w_i \geqq w_\infty \end{aligned}\right\} \tag{53}$$

A decrease of β therefore also acts favorably on w_*. For extreme values of $w_i : w_\infty$, $w_{max} = w_*$ is, of course, no longer valid, and the range of validity of the last equation diminished with decreasing β. The greatest increase of speed grows, in any case, when β decreases. The extent to which β is to be reduced is a matter that must be decided in each case individually.

III - EXAMPLE TO II, 3

It concerns Ruden's diffuser II (reference 2) with constant increase of speed $w_{max} = 1.2\ w_\infty$ along the nose contour in normal operating condition $\frac{w_i}{w_\infty} = 0.4$. By (36) and (38), in this case $\beta = \frac{11}{61} = 0.18$, $t_1 = \frac{488}{605} = 0.807$ and $t_2 = \frac{122}{5} = 24.4$, so that the upper contour of the nose is represented by

$$x = \tau - \log \tau$$

$$y = 0.18\sqrt{(\tau - 0.807)(24.4 - \tau)} + 4.55 \text{ arc tan } \frac{\sqrt{\tau - 0.807}}{\sqrt{24.4 - \tau}}$$

$$- 1.6 \text{ arc tan } \frac{11\sqrt{\tau - 0.807}}{2\sqrt{24.4 - \tau}}$$

$h = 1.8\pi = 5.655$ = one-half inside width.

Figures 11 to 14 give a comparison of the computed and measured pressure distributions for four different $\frac{w_i}{w_\infty}$ ratios. The ordinate $\frac{p_{st}}{q}$ is plotted against the abscissa x, (as in reference 2) that is, the ratio of static pressure $p - p_\infty$ to kinetic energy at infinity, $q = \frac{\rho}{2}{w_\infty}^2$. By Bernoulli's equation

$$p + \frac{\rho}{2} w^2 = p_\infty + \frac{\rho}{2} {w_\infty}^2$$

that is,

$$\frac{p - p_\infty}{\frac{\rho}{2} {w_\infty}^2} = 1 - \left(\frac{w}{w_\infty}\right)^2$$

The quotient $\frac{w}{w_\infty}$ is defined by the graphical method represented in I, 5. The agreement between theory and test is good. The larger discrepancies at the diffuser exit are explainable by the effect of the blunt end, while defects in workmanship are certainly noticeable at the mouth of the diffuser. The only appreciable point of difference is that the marked low-pressure peaks at the mouth are blunted by friction effects as postulated by theory for very high and very low $\frac{w_1}{w_\infty}$. The exact value of the maximum low pressure was, of course, not measured, but the measurements give a fairly good idea of the pressure distribution along the mouth of the diffuser. According to theory the maximum low pressure, that is, the maximum velocity of the flow, lies always at one of the end points of the nose contour. The velocities at these points, each divided by the flight speed, are represented in figure 15 for variable $\frac{w_1}{w_\infty}$; w_{max} is the greater of the two. The favorable effective range, filling the interval from 0.4 to 0.66, is plainly visible. The dashed line represents the ratio $\frac{w_{max}}{w_1}$ for $w_1 > w_\infty$. Continued toward the right it reaches the height of 9.3.

IV - EXAMPLE TO II, 4

Three different types of diffusers for unlimitedly variable $\frac{w_1}{w_\infty}$ were discussed, but the first two were rejected because of excessively high increase of speed even with great wall thickness. The third type depends only on one parameter β. The values t_1 and t_2 for given β are defined by (48) and (49). But, as stated previously after t_1 and t_2 are defined, β can be subsequently made variable again, so as to provide more favorable conditions for the operating range of principal interest by foregoing extreme values of $\frac{w_1}{w_\infty}$. Suppose the start is made from $\beta = 0.5$, for which $t_1 = \frac{1}{7}$ and $t_2 = 7$. The sum $t_1 + t_2$ has the value 7.14. This value is replaced by the mathematically convenient value

$$t_1 + t_2 = 8 \tag{54}$$

which is equivalent to a slight decrease of the initial value for β; this is now 0.47. With

$$t_1 = 4 - \sqrt{15} = 0.127$$

and

$$t_2 = 4 + \sqrt{15} = 7.873$$

to be computed from (54) and (49) the equations (17) for computing the diffuser contours are set up:

$$x = \tau - \log \tau$$

$$y = \beta\left[\sqrt{8\tau - \tau^2 - 1} + 8 \text{ arc tan } \frac{\tau - 0.127}{\sqrt{8\tau - \tau^2 - 1}} - 2 \text{ arc tan } \frac{7.873(\tau - 0.127)}{\sqrt{8\tau - \tau^2 - 1}}\right]$$

$$h = (1 + \beta)\pi$$

In accord with previous studies, the values chosen for β are below the initial value 0.47. Figure 16 shows the contours for $\beta = 0.1$, 0.2, 0.3, and 0.4. Scale variations ensure that all diffusers have equal absolute wall thickness d. The values of the relative thickness $\delta = \frac{d}{h}$ are also indicated.

Figure 17 shows the maximum speed distribution for the four diffusers plotted against $\frac{w_i}{w_\infty}$, which is, for $w_i < w_\infty$ the ratio $\frac{w_{max}}{w_\infty}$, for $w_i > w_\infty$ the ratio $\frac{w_{max}}{w_i}$. A decrease of β in the central range is favorable, at the ends, unfavorable. The others refer to the case $\beta = 0.4$. Figures 18 to 20 represent the relative pressure distributions for different operating conditions in comparison with the pressure-distribution curves of figures 11 to 14.

V - APPENDIX

1.- Variation of the Function $\left|\frac{dz}{d\zeta}\right|$ of t

Writing x, x_1, x_2, and y instead of t, t_1, t_2, and $\left|\frac{dz}{d\zeta}\right|$ so as to conform with the conventional notations of analytical geometry, the equation of the central arc of C reads

$$y^2 = (x + 1)^2 - \beta^2(x + x_1)(x + x_2)$$

or

$$x^2(1 - \beta^2) - y^2 + x\left[2 - \beta^2(x_1 + x_2)\right] + 1 - \beta^2 x_1 x_2 = 0$$

If a curve of the second order

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0$$

is to be analyzed, the determinants

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad (a_{ik} = a_{ki})$$

and

$$A_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

must be borne in mind. The decomposition of the curve in a pair of straight lines is indicated by $A = 0$, while for $A \neq 0$

$$A_{33} > 0 \quad \text{gives an ellipse}$$

$$A_{33} = 0 \quad \text{a parabola}$$

$A_{33} < 0$ a hyperbola. In the present case

$$A = \begin{vmatrix} 1 - \beta^2 & 0 & 1 - \beta^2 \dfrac{x_1 + x_2}{2} \\ 0 & -1 & 0 \\ 1 - \beta^2 \dfrac{x_1 + x_2}{2} & 0 & 1 - \beta^2 x_1 x_2 \end{vmatrix} = \left(1 - \beta^2 \frac{x_1 + x_2}{2}\right)^2 - (1 - \beta^2)(1 - \beta^2 x_1 x_2)$$

$$= \beta^2(1 + x_1 x_2 - x_1 - x_2) + \beta^4\left(\frac{x_2 - x_1}{2}\right)^2$$

$$= \left(\frac{x_2 - x_1}{2}\right)^2 \beta^2(\beta^2 - \beta_o^2)$$

with

$$\beta_o = \frac{2\sqrt{(1 - x_1)(x_2 - 1)}}{(x_2 - x_1)}$$

Decomposition therefore occurs for $\beta = 0$ and $\beta = \beta_o$, where, since $1 - \beta_o^2 = \dfrac{(x_1 + x_2 - 2)^2}{(x_2 - x_1)^2}$ is certainly not greater than unity and attains unity only for $x_1 + x_2 = 2$; A_{33} has the value $\beta^2 - 1$, hence, hyperbolas when $\beta < 1$, ellipses when $\beta > 1$. The case $\beta = 1$ results in a parabola, or, for $x_1 + x_2 = 2$ in a pair of parabolas, of which only the one straight line above the abscissa axis is in evidence.

2 - General Determination of w_{max}

Elsewhere w_{max} had been defined geometrically. For the case that w_{max} is assumed at an inside point of the nose contour, a tangent is placed from a given point of the t-axis on the conical section to which the central arc of C relates. The corresponding calculation is as follows: The equation of the conical section is

$$(t + 1)^2 - \beta^2(t + t_1)(t + t_2) - s^2 = 0$$

or

$$(1 - \beta^2)\ t^2 + \left[2 - \beta^2(t_1 + t_2)\right] t + 1 - \beta^2 t_1 t_2 - s^2 = 0$$

while the considered point has the coordinates $t = -t_o$, $s = 0$. For computing the tangents from point (x_o, y_o) on the conical section

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0$$

the "polar" of (x_o, y_o) with respect to the conical section

$$a_{11}xx_o + a_{12}(xy_o + yx_o) + a_{22}yy_o + a_{13}(x + x_o) + a_{23}(y + y_o) + a_{33} = 0$$

are determined.

This straight line dissects the two constant points on the conical section. In the present case the equation of the polar is

$$-(1 - \beta^2)\ tt_o + \left[1 - \beta^2 \frac{t_1 + t_2}{2}\right](t - t_o) + (1 - \beta^2\ t_1 t_2) = 0$$

or

$$t = -\frac{\left(1 - \beta^2 \frac{t_1 + t_2}{2}\right)t_o - (1 - \beta^2\ t_1 t_2)}{(1 - \beta^2)\ t_o - \left(1 - \beta^2 \frac{t_1 + t_2}{2}\right)} \tag{55}$$

The polar is normal to the t-axis. Denoting the value given by (55) with $-t_o'$, gives

$$\frac{w_{max}}{w_\infty} = \frac{|t_o - t_o'|}{s(-t_o')} (1 + \beta)$$

that is

$$\left(\frac{w_{max}}{w_\infty}\right)^2 = (1 + \beta)^2 \frac{(t_o - t_o')^2}{s^2(-t_o')}$$

A simple calculation gives:

$$\left(\frac{w_{max}}{w_\infty}\right)^2 = \frac{4(1 + \beta)^2}{(t_2 - t_1)^2 \beta^2(\beta_o^2 - \beta^2)} \left[(1 - t_o)^2 - \beta^2(t_1 - t_o)(t_2 - t_o)\right] \tag{56}$$

β_o is the value given by (20); β is smaller than β_o. The formula holds for the case that the value t_2 ranges between t_1 and t_2, hence for

$$t_1 \leqq \frac{\left(1 - \beta^2 \dfrac{t_1 + t_2}{2}\right)t_o - \left(1 - \beta^2 t_1 t_2\right)}{(1 - \beta^2)\ t_o - \left(1 - \beta^2 \dfrac{t_1 + t_2}{2}\right)} \leqq t_2$$

Determine next the two t_o values for which $t_o' = t_1$ or $t_o' = t_2$. These values are given by

$$t_3 = 1 + \frac{\dfrac{\beta^2}{2}(t_2 - t_1)(1 - t_1)}{1 - t_1 - \dfrac{\beta^2}{2}(t_2 - t_1)}$$

and

$$t_4 = 1 - \frac{\dfrac{\beta^2}{2}(t_2 - t_1)(t_2 - 1)}{t_2 - 1 - \dfrac{\beta^2}{2}(t_2 - t_1)} \qquad \text{(Figure 21)}$$

For

$$\beta^2 < \frac{2(1 - t_1)}{t_2 - t_1}$$

t_3 ranges between 1 and ∞, for

$$\beta^2 < \frac{2(t_2 - 1)}{t_2(t_2 - t_1)}$$

t_4 lies between 0 and 1. If β satisfies these two inequalities, formula (56) holds for all t_o that meet the conditions $0 \leqq t_o \leqq t_4$ or $t_o \geqq t_3$, while $w_{max} = w_*$ for $t_4 \leqq t_o \leqq t_3$. If only one of the inequalities for β is satisfied, there is only one validity interval for (56), and if none of the inequalities applies, $w_{max} = w_*$.

PART II

I - AUXILIARY MATHEMATICAL EXAMINATION

Consider the conformal mapping effected by $w = z + \frac{1}{z}$ in such a way that z and w are made to vary in the same plane, z being restricted to values which, geometrically speaking, lie outside of the unit circle and above the real axis. The case then presents a conformal mapping of a semicircularly notched half plane on the entire half plane.

It is seen at once that

(a) each inside point is shifted nearer to the real axis by the mapping

(b) the boundary points lying on the real axis all travel outward, that is, for $z \geqq 1$ toward the right, for $z \leqq -1$ toward the left,

(c) every boundary interval lying on the real axis is reduced in its length.

These three facts hold, as will be shown, not only for the special mapping according to $z + \frac{1}{z}$, but very generally for the conformal mapping on a half plane provided with any desired notch on the full half plane. More exactly the following is valid: (fig. 22)

Let B represent a simple connected space in the upper half of the z-plane bounded by the two semistraight lines $z \leqq a$ and $z \geqq b$ (a and b real, $a < b$) and a curved arc free of double points connecting the points a and b and with exception of the end points running entirely within the upper half plane; B is mapped by the function $w = f(z)$ on the whole upper half plane and the mapping function existing according to Riemann's mapping principle is so standardized that the infinitely remote point transforms in the finitely remote point and that in the development

$w = Cz + c_0 + \frac{c_1}{z} + \frac{c_2}{z^2} + \ldots \; (C > 0)$ applicable to the vicinity of infinite distance, the coefficients C and c_0 attain the values 1 and 0 (standardization at rest in infinity, $w = z + ((0))$).

In this instance

(a) $J(w) < J(z)$ for all inside points

(b) $w > z$ for $z \geqq b$, $w < z$ for $z \leqq a$

(c) $|w_2 - w_1| < |z_2 - z_1|$ for $z_2 > z_1 \geqq b$ and for $z_1 < z_2 \leqq a$ $(w_1 = f(z_1))$

Proof:

For the first it is noted that the function $w(z)$ can be analytically continued by the reflection principle beyond the real axis. Putting $z = x + iy$,

$$w - z = w' - u' + iv'$$

and considering v' as function of x and y, v' is a regular potential function in the entire space B, which on approaching the infinitely remote point disappears. Along the boundary piece on the real axis $v' = 0$, along the rest of the boundary $v' < 0$. Because of the regular behavior at infinity the validity of $v' < 0$ on the inside can be deduced from the inequality $v' \leqq 0$ at the boundary. Since $v' = J(w) - J(z)$ the claim (a) is therefore proved.

Let $(x_o, 0)$ represent a boundary point of B on the real axis. As $v'(x_o, 0) = 0$ and $v'(x_o, y) < 0$ for sufficiently small positive y values $\frac{\partial v'}{\partial y}(x_o, 0)$ is certainly $\leqq 0$.

The equal sign must be excluded, for if $\frac{\partial v'}{\partial y}(x_o, 0)$ were equal to 0, then $\frac{\partial v'}{\partial x}$ as well as $\frac{\partial v'}{\partial y}$ would vanish in point $(x_o, 0)$, that is, the derivative $\frac{dw'}{dz}$ would be 0 at the particular place and the development of w' would assume the form

$$w' = \alpha_o + \alpha_\kappa (z - x_o)^\kappa + \ldots$$

with

$$\kappa \geqq 2, \ \alpha_\kappa \neq 0 \ (\alpha_o \text{ real})$$

But then to a small two-dimensional surrounding of x_0 there would correspond a multi-lobed surrounding of point α_0 through the mapping according to $w'(z)$, and v' would have to assume positive values in the upper half plane, which cannot be, according to (a). By the Cauchy-Riemann differential equations $\frac{\partial u'}{\partial x}$ is < 0 for all boundary points of B lying on the real axis, that is, u' decreases when traveling from left to right. As to the disappearance of u' at infinity it is seen that u' must be positive for $z \geqq b$, whereas u' assumes negative values for $z \leqq a$. But along the real axis $u' = w - z$, whence the claim (b).

Lastly, for $z_2 > z_1 \geqq b$ and for $z_1 < z_2 \leqq a$:

$$(w_2 - w_1) - (z_2 - z_1) = (w_2 - z_2) - (w_1 - z_1) = u'_2 - u'_1$$

This monotonic behavior of u' proves the correctness of c.

II - MATHEMATICAL TREATMENT OF GENERAL INLET DIFFUSERS

The following investigation contains four complex variables all of which are in analytical relationship to each other. The first variable, z, is the complex coordinate in the plane of the diffuser, the second, W, is given by the value of the stream function, the third, w, by the value of the complex velocity, and the fourth, t, is a pure mathematically explained auxiliary quantity. The range of variation of z is represented in figure 23(a). The diffuser is assumed to be of infinite length; it is to be symmetrical and bounded by two convex curves with constant direction of tangents, whose pieces extending to infinity are rectilinear from a certain point on. The convexity required for the outside is natural; but at the inside wall the limitation to convex forms imposes a relinguishment not shown beforehand by physical-technical considerations. On the contrary, the constructed engine cowlings used in practice for $w_i < w_\infty$ rather exhibit cross-sectional enlargement downstream from the diffuser opening, that is, no convex contours (fig. 23(b)). Nevertheless the limitation is restricted to convex forms for the reason that the mathematical analysis affords a simple connection between quantity $\delta = \frac{d}{h}$ (figs. 23(a) and 23(b)) and the velocity distribution along the contour; but the quantity δ is decisive only for convex forms in problems of engine installation, while on forms with cross-sectional enlargement the quantity $\delta_* = \frac{d*}{h*}$ is decisive, the dependence of which on the velocity distribution is far more complicated.

Only one flow which corresponds to a specific ratio $\frac{w_i}{w_\infty}$ is investigated. It is represented by an analytical function $W = F(z)$. Putting $W = U + iV$, the streamlines are given by V = constant; $w = \frac{dF(z)}{dx}$ is the complex velocity whose conjugate complex value represents the actual velocity in magnitude and direction. The function $F(z)$ can be replaced by $cW + c'$ with positive c and any complex c' without altering the flow pattern, because only a multiplication of the complex velocity by a constant positive factor is effected, which is equivalent to a variation in mass unit.

By $W = F(z)$ the z space is mapped on a symmetrical slot region which is, the slots run parallel to the real axis rightward to infinity. By taking advantage of the previously cited freedom in the determination of W, it can be assumed that the two slots are given by $V = \pm\pi$, $U \geqq 0$ (fig. 24).

The quantity $w = \frac{dF(z)}{dz}$ varies in a symmetrical space lying entirely within finite limits. Putting $w = \rho e^{i\tau}$, the actual velocity is indicated by $\rho e^{-i\tau}$. The boundary values of τ are given directly by the diffuser contours, $-\tau = \arg dz$, where, it is true, the orientation of the line element still remains questionable, that is, τ is defined up to multiples of π. In any case it follows from the assumed convexity of the contours that τ varies monotonically, if the contour is followed from the stagnation point to infinity in one or the other direction. The further study is restricted to the upper half of the z space. The image of this half space in the w-plane is bounded by a length lying on the real axis, which corresponds to the line of symmetry of the diffuser and the two straight walls, and by a curve arising from the curved part of the diffuser contour. The curve proceeds entirely in the upper w-plane when the stagnation point lies on the outside diffuser wall; it runs in the lower half plane, when the stagnation point lies on the inside wall; it splits into two arcs, each belonging to a half plane, when the stagnation point is a point of the curved contour. In the latter case the two branches of the curves meet in the zero point of the w-plane and have a common tangent for $w = 0$. In any event the curved part of the boundary of the w-space is free from double points because of the monotonic variation of τ; from this it can be concluded without difficulty that the w-space must be a smooth, that is, single lobed, space. The figure 25 represents potential space forms. The zero point is always a boundary point. The w_i and w_∞ indicate the velocity inside the diffuser and the flight speed.

When the w-space with the points w_1 and w_∞ is given, the diffuser design is defined. For the w-space is mapped on the W-space of which only the upper half is taken into account; $w = 0$ changes to $W = \pi i$, while to w_1 and w_∞ correspond the two infinitely distant boundary points of the w-space. The invariably existing mapping function $W(w)$ enables z to be computed as function of w.

Since

$$w = \frac{dW}{dz}$$

$$z = \int \frac{dW}{w} = \int \frac{1}{w} \frac{dW}{dw} dw$$

According to it z appears to be determined up to an additive constant. The other possible similarity mappings in the z-plane follow when it is remembered that the quantity W is arbitrarily normalized by the foregoing, and that truly directional similarity mapping with 0 as central point in the w-plane is admissible.

Since pressure and velocity distribution are the chief factors governing the quality of the diffuser, the usual process is to proceed from the velocity patterns and to determine the contour according to it.

In the present report the hodographic method is resorted to. But, while in reference (1) the amount of the w-space was automatically restricted by the assumption that the curved part of the contour is a half circle, no special assumption is made here.

It is convenient to replace the W-space by a half plane, putting $W = t - \log t + \pi i - 1$. It is readily verified that by this relation between t and W the W-space is, by appropriate fixing of the logarithm, mapped on the upper half of the t-plane. The two infinitely distant points of the W-space correspond to the points $t = 0$ and $t = \infty$, while $W = \pi i$ changes to $t = 1$ (fig. 26).

The w-space must accordingly be mapped on the upper half plane by means of a function $t(w)$, the points $0, w_1, w_\infty$ must change into $1, 0, \infty$, (fundamental mapping of w-space with boundary normalizing). The corresponding diffuser form is then defined by

$$z = \int \frac{1}{w} \frac{dW}{dw} \, dw = \int \frac{1}{w}\left(1 - \frac{1}{t}\right) \frac{dt}{dw} \, dw$$

III - A GENERAL THICKNESS FORMULA

The main object of the present note is to prove the fact that for fixed values of w_1, w_∞ and w_{max} the corresponding wall thickness of the diffuser cannot become less than the value computed by Ruden for the case of constant increase of velocity along the nose

$$\frac{(w_\infty - w_1)^2}{w_{max}^2 - w_\infty^2}$$

only Ruden's diffusers reach the minimum value.

To be able to compute the quantity δ for any desired w-space only the behavior of the function $z(w)$ near w_∞ needs to be known. The quantity t becomes infinite for w_∞ in first order and has a development of the form

$$t = \frac{f}{w - w_\infty} + \text{reg.}$$

with f positive for $w_1 < w_\infty$ and negative for $w_1 > w_\infty$. It is to be noted that $t(w)$ over an interval of the real axis containing the point w_∞ on the inside certainly can be continued by the reflection principle.

Accordingly

$$t = \frac{f}{w - w_\infty}\left(1 + ((0))\right)$$

$$\frac{1}{t} = \frac{w - w_\infty}{f}\left(1 + ((0))\right)$$

$$\frac{dt}{dw} = -\frac{f}{(w - w_\infty)^2} + \text{reg.}$$

$$\left(1 - \frac{1}{t}\right)\frac{dt}{dw} = -\frac{f}{(w - w_\infty)^2} + \frac{1}{w - w_\infty} + \text{reg.}$$

where reg. indicates a function regular for w_∞, $((0))$ one regular for w_∞ and disappearing in w_∞ itself. Adding the development of $\frac{1}{w}$

$$\frac{1}{w} = \frac{1}{w_\infty} - \frac{1}{w_\infty^2}(w - w_\infty) + \ldots$$

gives then

$$z = \int\left[\frac{1}{w_\infty} - \frac{1}{w_\infty^2}(w - w_\infty) + \ldots\right]\left[-\frac{f}{(w - w_\infty)^2} + \frac{1}{w - w_\infty} + \text{reg}\right]dw$$

$$= \frac{f}{w_\infty}\frac{1}{w - w_\infty} + \left(\frac{f}{w_\infty^2} + \frac{1}{w_\infty}\right)\log(w - w_\infty) + \text{reg.}$$

From this formula the amount of $d + h$, that is, the variation of the imaginary component of z on passing through w_∞, can be taken at once

$$d + h = \pi\left(\frac{f}{w_\infty^2} + \frac{1}{w_\infty}\right)$$

Quantity h is correspondingly given by the variation of the imaginary component of z on passing through w_i. In this instance

$$t = c(w - w_i) + \ldots \quad c \neq 0$$

$$\frac{1}{t} = \frac{1}{c}\,\frac{1}{w - w_i} + \text{reg.}$$

$$\left(1 - \frac{1}{t}\right)\frac{dt}{dw} = -\frac{1}{w - w_i} + \text{reg.}$$

$$\frac{1}{w} = \frac{1}{w_i} + ((0))$$

$$z = \int\left[\frac{1}{w_i} + ((0))\right]\left[-\frac{1}{w - w_i} + \text{reg.}\right]dw = -\frac{1}{w_i}\log\,(w - w_i) + \text{reg.}$$

that is, $h = \dfrac{\pi}{w_i}$; hence

$$\frac{d + h}{h} = \delta + 1 = \frac{w_i}{w_\infty^{\ 2}}\,(f + w_\infty)$$

and

$$\delta = \frac{1}{w_\infty^{\ 2}}\,(w_i\,f + w_i w_\infty - w_\infty^{\ 2})$$

While in the formulas for h and d + h the arbitrary standardization of W plays a part, the formula for δ is of completely general validity. The quantity f, that is, the residuum of t in point w_∞ has the dimension of a velocity. At fixed values of w_i and w_∞, δ is solely dependent on f.

The thickness formula is now used to compute the wall thickness of a Ruden type of inlet diffuser. The w-space is a half circle; the center is the zero point, the radius is w_{max} and for $w_i < w_\infty$ the half circle lies in the lower, for $w_i > w$ in the upper half plane. The mapping function t(w) is a rational function of the second degree, with zero points at w_i and $\dfrac{w_{max}^2}{w_i}$, while assuming the value ∞ for w_∞

and $\dfrac{w_{max}^2}{w_\infty}$

Accordingly

$$t = \text{constant}\ \frac{(w - w_i)\left(w - \dfrac{w_{max}^2}{w_i}\right)}{(w - w_\infty)\left(w - \dfrac{w_{max}^2}{w_\infty}\right)}$$

and the constant factor gives the value 1, since $t(0) = 1$. The residuum for w_∞ is

$$f = \frac{(w_\infty - w_i)\left(w_\infty - \dfrac{w_{max}^2}{w_i}\right)}{w_\infty - \dfrac{w_{max}^2}{w_\infty}} = (w_\infty - w_i)\ \frac{(w_{max}^2 - w_\infty w_i)\ w_\infty}{(w_{max}^2 - w_\infty^2)\ w_i}$$

hence

$$\delta = \frac{1}{w_\infty}\left[(w_\infty - w_i)\ \frac{w_{max}^2 - w_\infty w_i}{w_{max}^2 - w_\infty^2} + w_i - w_\infty\right] = \frac{(w_\infty - w_i)^2}{w_{max}^2 - w_\infty^2}$$

IV - PROOF OF THE MINIMUM FORMULA

A piece is cut out of the w-space B of figure 25(a) ($w_i < w_\infty$, stagnation point at inside wall), as indicated in figure 27(a). What is the variation experienced by δ? According to the cited thickness formula it is sufficient to analyze the variation of f. It is $f > 0$.

The reduced space is denoted by B'. The function $t(w)$ that maps the original space on the half plane is regarded as known. The mapping function $t'(w)$ for B' is then obtained in the following manner: first map B' by means of $t(w)$; the result is a half plane with a notch. This is then mapped by means of a function $t''(t)$ on a complete half plane, the mapping to be standardized at infinity according to the formula $t'' = t + ((0))$; t' is then a whole linear formation of t'', $t' = \lambda t'' + \mu$. Hence, since t' as function of t at infinity has a development $t' = \lambda t + \mu + \dfrac{c_1}{t} + \dfrac{c_2}{t^2} \ldots$, the relation $f' = \lambda f$ holds for the residues f and f'. Quantity λ is defined by the condition that points 0 and w_i shall change to $t' = 1$ and $t' = 0$. Hence $t(0) = 1$ and $t(w_i) = 0$. At transition from t to t''. the interval (0,1) is reduced in any event, since the notch lies outside the interval. Accordingly λ must definitely be greater than unity, hence

$$f' > f$$

The wall thickness therefore increases as the w-space is reduced. When the w-space increases, δ decreases accordingly. So, if a specific w_{max} is given, which may be attained but not exceeded, the w-space may not extend beyond the circular space $|w| \leqq w_{max}$. The wall thickness is a minimum when the space available is completely utilized, that is, when the amount of the velocity along the entire nose is made equal to w_{max}. This proves the minimum theorem for $w_i < w_\infty$ when assuming the stagnation point located at the inside wall.

Next, the stagnation point is placed on the curved portion of the contour, while $w_i < w_\infty$ (fig. 27(b)). In this instance any increase of the w-space in the lower half plane is again associated with a decrease in δ. This decrease in δ can be continued further by reducing the part of the w-space situated in the upper half plane. This is readily verified by the mapping $t'(w)$ of the reduced space as above, passing through $t(w)$ to a half plane with a notch, then completing the transition to $t'' = t + ((0))$ and to $t' = \lambda t + \mu$.

The difference now consists in the notch in the t-plane lying between 0 and 1, thus increasing the interval (0, 1) to t'' on transition. Therefore $\lambda < 1$ and $f' < f$, that is δ becomes smaller as claimed. The entire piece of the w-space lying in the upper half plane is omitted, wherewith the stagnation point shifts to the inside wall.

The optimum within the group of convex diffusers for specific w_{max} is therefore actually reached by Ruden's inlet diffuser.

If $w_i > w_\infty$ the proof is entirely analogous. The residuum f is then negative. A notch in the w-space of the upper half plane appears then in the t-plane between 0 and 1 and results in $\lambda < 1$. Accordingly $f' = \lambda f > f$, and δ increases. It is seen further that a piece of the w-space lying in the lower half plane must be reduced, in order that δ decrease. The optimum w-space for a given w_{max} is therefore the half circle $|w| \leqq w_{max}$, $J(w) \geqq 0$.

V. CONCLUDING REMARKS

The subsequent discussion is limited to the case $w_i < w_\infty$. As already stated, the diffusers used in practice exhibit cross-sectional enlargements downstream from the diffuser orifice (fig. 23(b)). The corresponding hodograph record is given in figure 28, it being assumed that the stagnation point lies on the inside wall. The quantity $\delta = \frac{d}{h}$ can be reduced below Ruden's value, if the hodographic picture is so chosen that, aside from the straight piece, the lower half of the circle $|w| = w_{max}$, and between 0 and w_i an arc running in the lower half plane takes part on the demarcation.

The nearer the last arc is pulled to the remaining part of the rim, the smaller δ becomes. The value $\delta = 0$ is attainable. However, two facts should be borne in mind.

First, the quantity governing the practicability of the diffuser is $\delta^* = \frac{d^*}{h^*}$, not $\delta = \frac{d}{h}$, so this quantity would have to be analyzed in relation to the w-space. The resulting relation might not be as simple as for δ. In any case it is doubtful whether δ^* varies in general, as δ, and to what value δ may be reduced.

Second, the study of inlet diffusers of constant internal cross section can be restricted to $w_i : w_\infty : w_{max}$ and δ, but not on the forms considered here. Ruden's diffusers indicate considerable regulating capacity and operate nearly without loss to the extent that the conversion of velocity to pressure is essentially effected upstream from the diffuser mouth. To forestall excessive variations the section enlargement must be done very carefully, that is, the hodographic pattern must not depart very much from Ruden's. In any case, constant increase of speed along the curved part of the outside contour must be specified for the design operating condition, and the smallness of the notch κ (fig. 28) vouches for the outside contour itself not deviating perceptably from that by Ruden. To this extent the minimum formula proved in this report is of practical significance in spite of the limitation to constant cross sections. Forms with cross-sectional enlargments are to be discussed in a subsequent report.

Translated by J. Vanier
National Advisory Committee
for Aeronautics

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1. Ruden, P.: Ebne Symmetrische Fangdiffusoren. Forschungsbericht Nr. 1209. (Available as NACA TM 1279.)

2. Ruden, P.: Windkanalmessungen an ebner Fangdiffusoren. Forschungsbericht Nr. 1325.

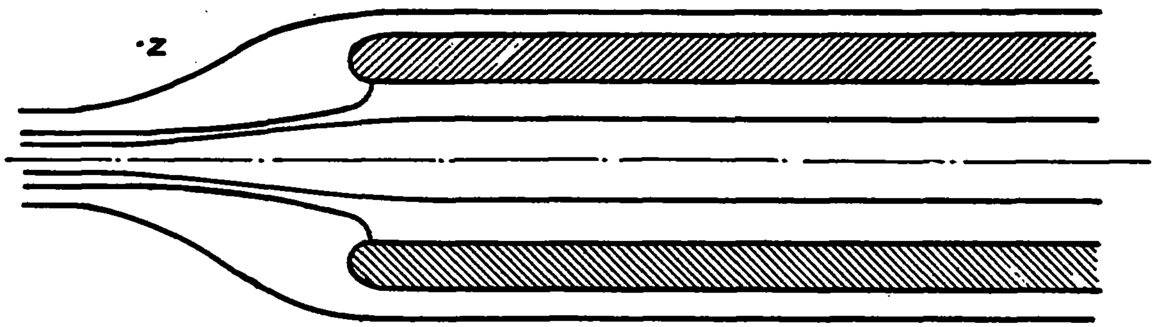


Figure 1.- Schematical representation of the diffuser contours and corresponding flow.

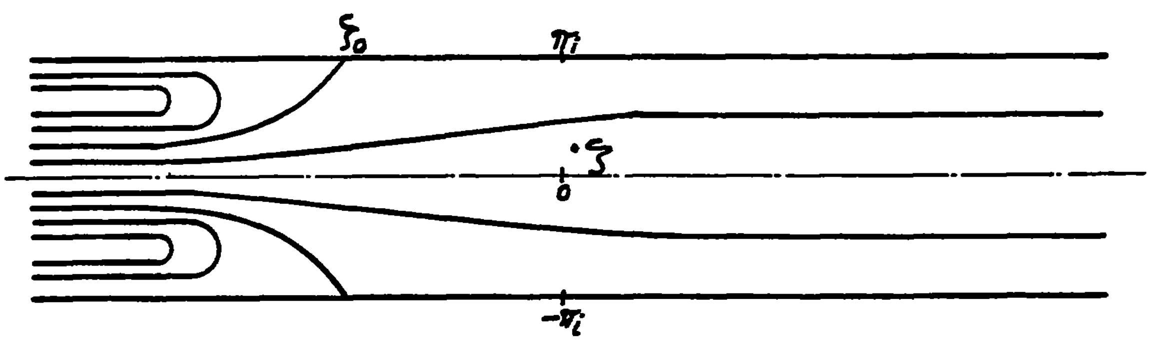


Figure 2.- Schematical representation of a flow in the parallel strip.

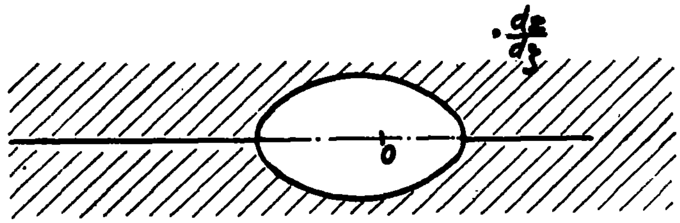


Figure 3.- By $dz/d\zeta$ the parallel strip is mapped to a region symmetrical with respect to the real axis bounded by an ellipse, a finite and an infinite piece of the real axis.

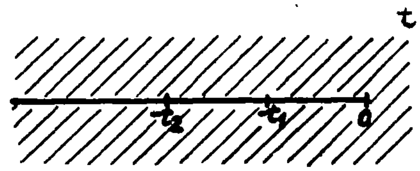


Figure 4.- By $t = e^{-\zeta}$ the parallel strip is mapped to a plane that is cut along the negative half of the real axis. The two boundary pieces corresponding to the interval $(-t_2, -t_1)$ will later yield the curved parts of the diffuser contour.

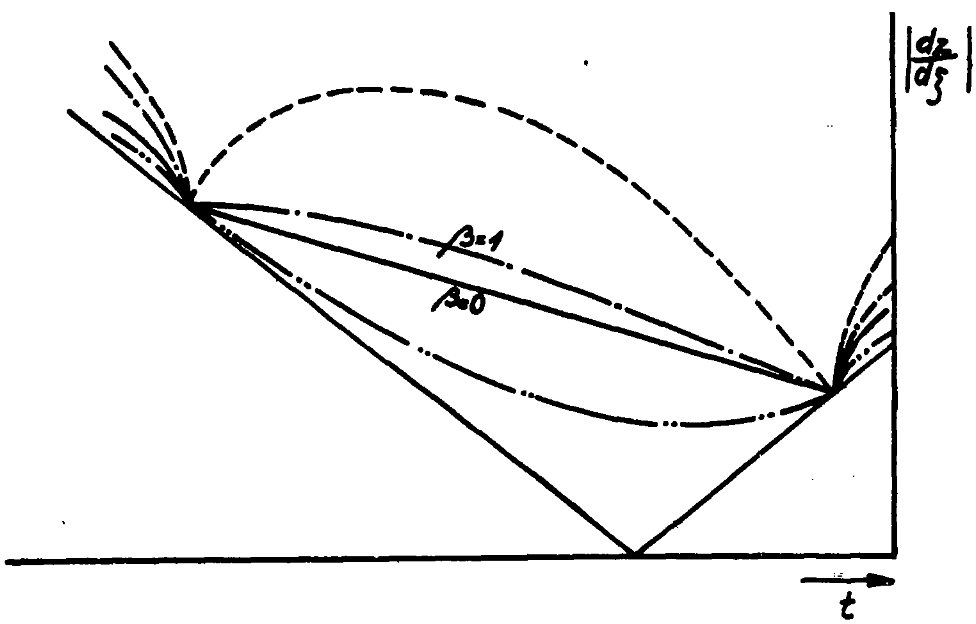


Figure 5.- $|dz/d\zeta|$ plotted against $t(t < 0)$ for various values of β (t_1 and t_2 fixed).

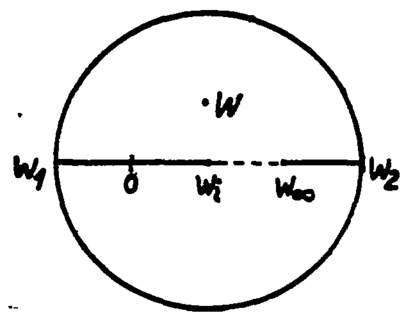


Figure 6.- The zone of variation of the complex velocity w according to P. Ruden.

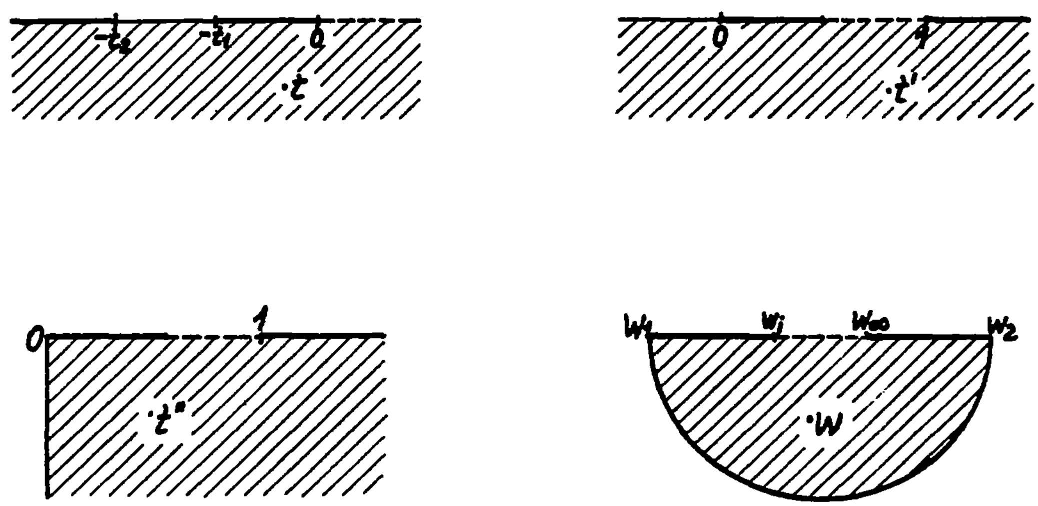


Figure 7.- The conformal mapping of the t-space on the w-space.

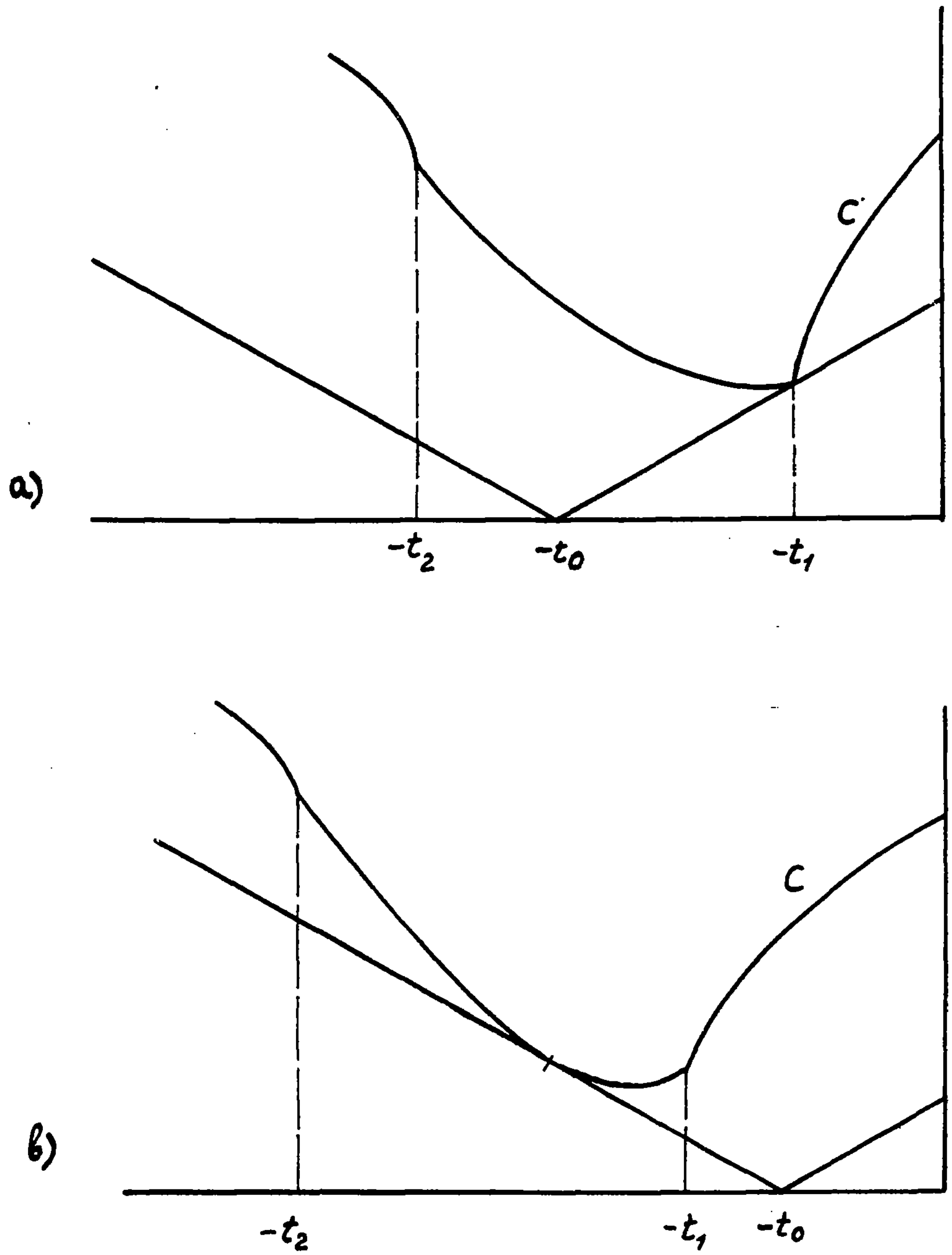


Figure 8a,b.-- Maximum velocity is reached either at a sharp bend of C(a) or at a point of the central arc (b).

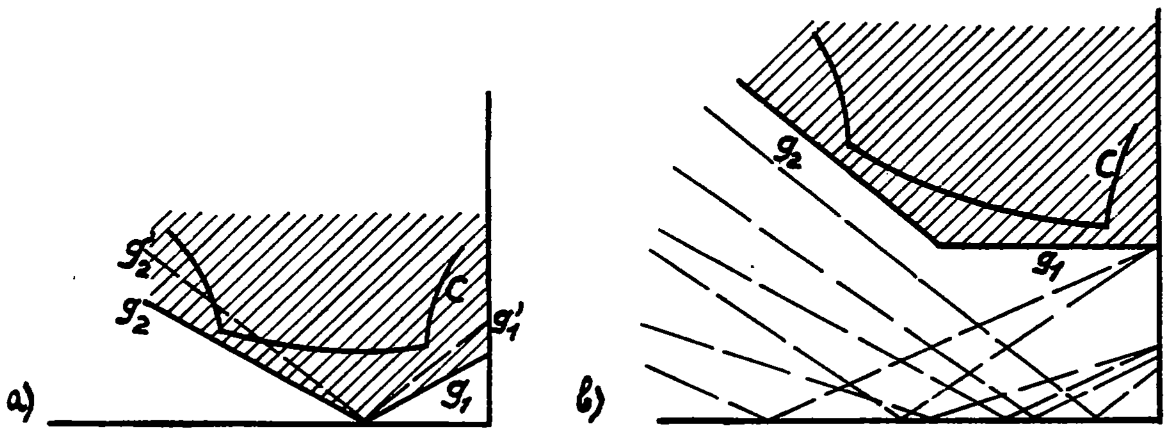


Figure 9a,b.- If w_{max} is to stay beneath a certain limit, the curve C must not approach the t-axis too closely. For fixed values of w_i and w_∞(a), as well as for variable values of w_i and w_∞(b), there always results a space bounded by three straight lines from which C must not emerge.

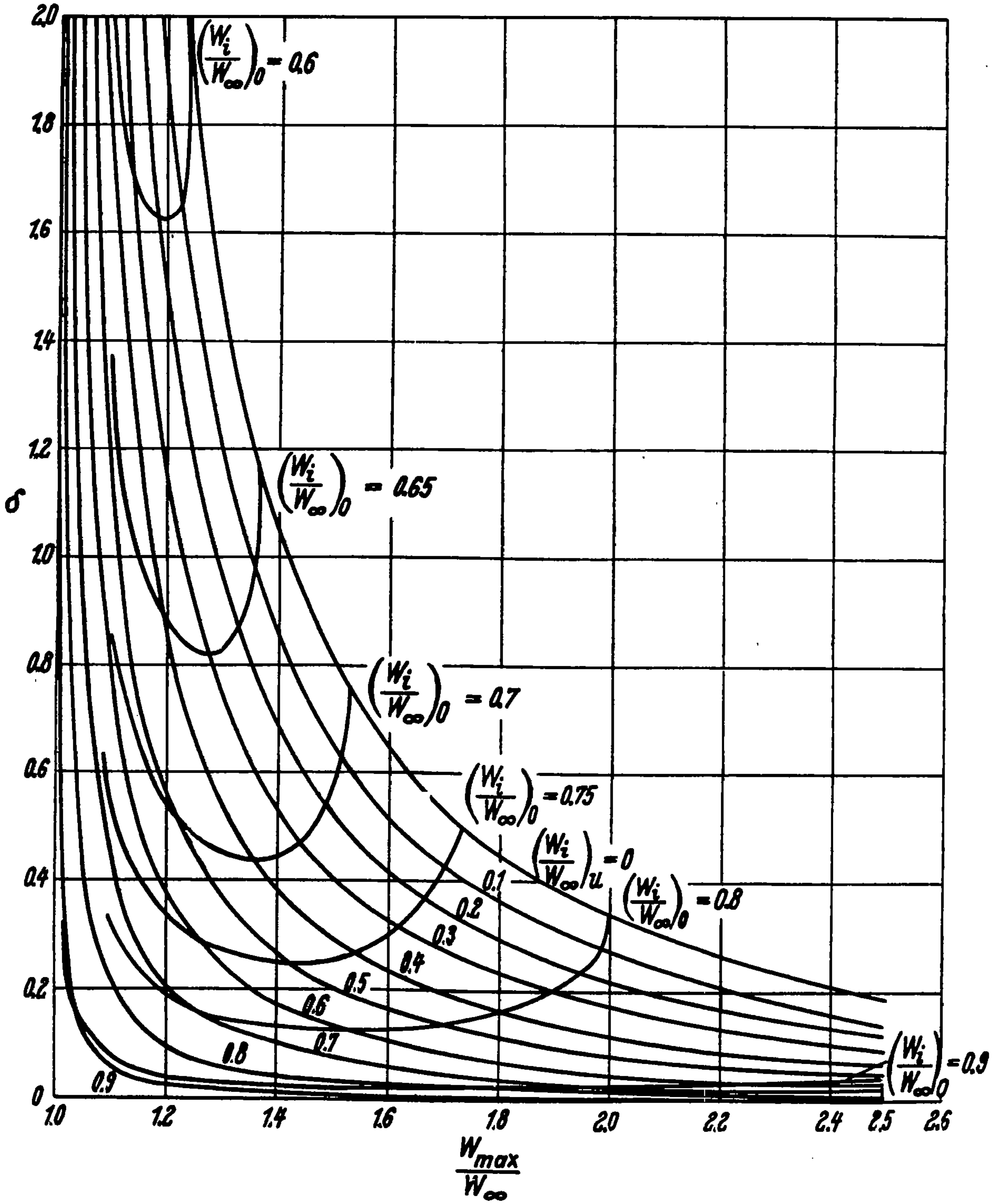


Figure 10.- The favorable effective range of a Ruden inlet diffuser is given by $\left(\frac{w_i}{w_\infty}\right)_u \leqq \frac{w_i}{w_\infty} \leqq \left(\frac{w_i}{w_\infty}\right)_o$. The figure represents the two boundaries as functions of w_{max}/w_∞ and δ. The curves $(w_i/w_\infty)_o$ = const. show in the neighborhood of $w_{max}/w_\infty = 1$ a variation similar to the curves $(w_i/w_\infty)_u$ = const. They are not fully drawn for reasons of clarity.

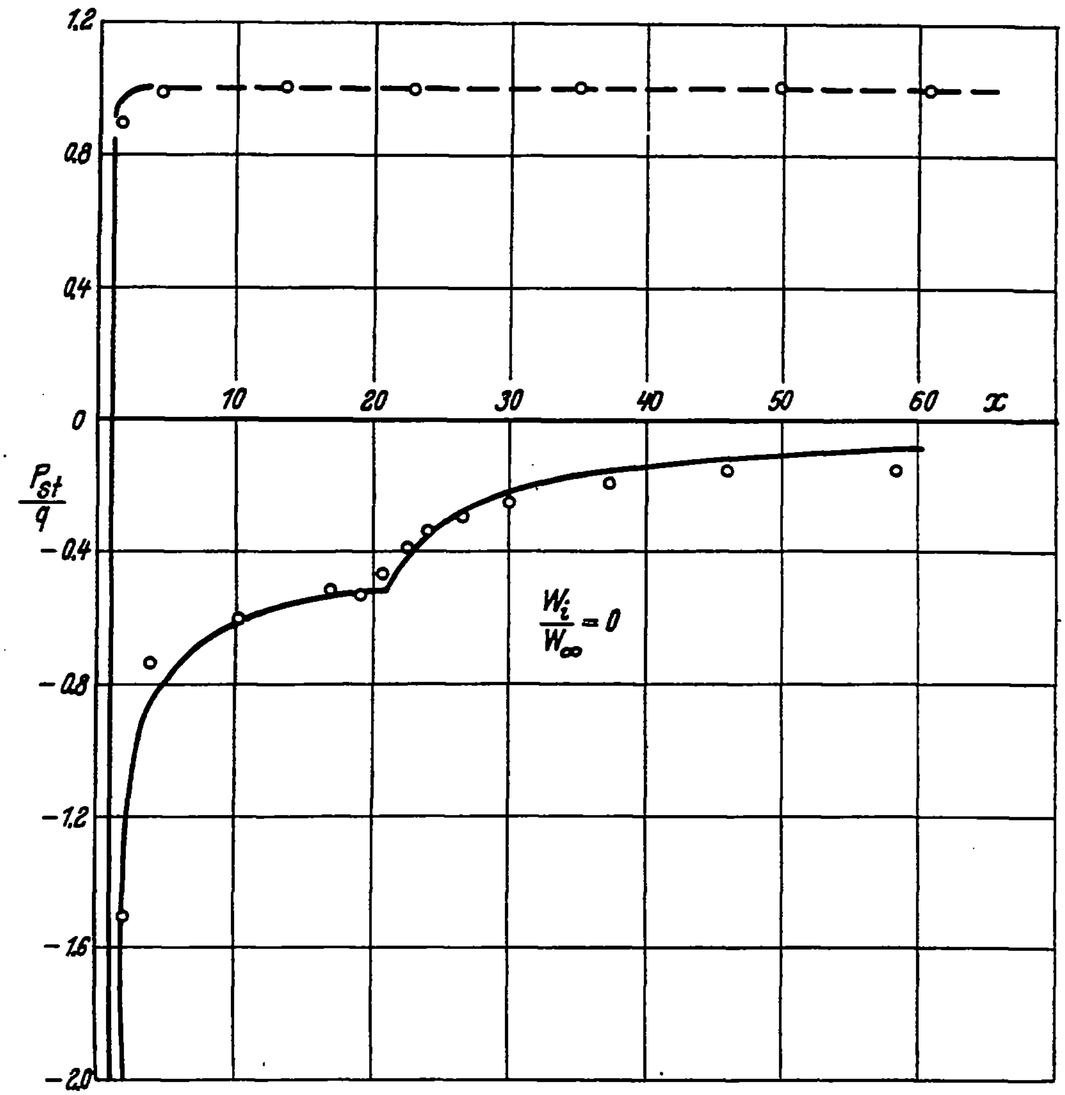


Figure 11

Figures 11-14.- Pressure-distribution curves for a Ruden inlet diffuser. Normal operating condition: $w_i/w_\infty = 0.4$; $w_{max}/w_\infty = 1.2$. The solid curves represent the theoretical pressure distribution along the outer wall, the dashed curves the theoretical pressure distribution along the inner wall. The circles correspond to the measured values.

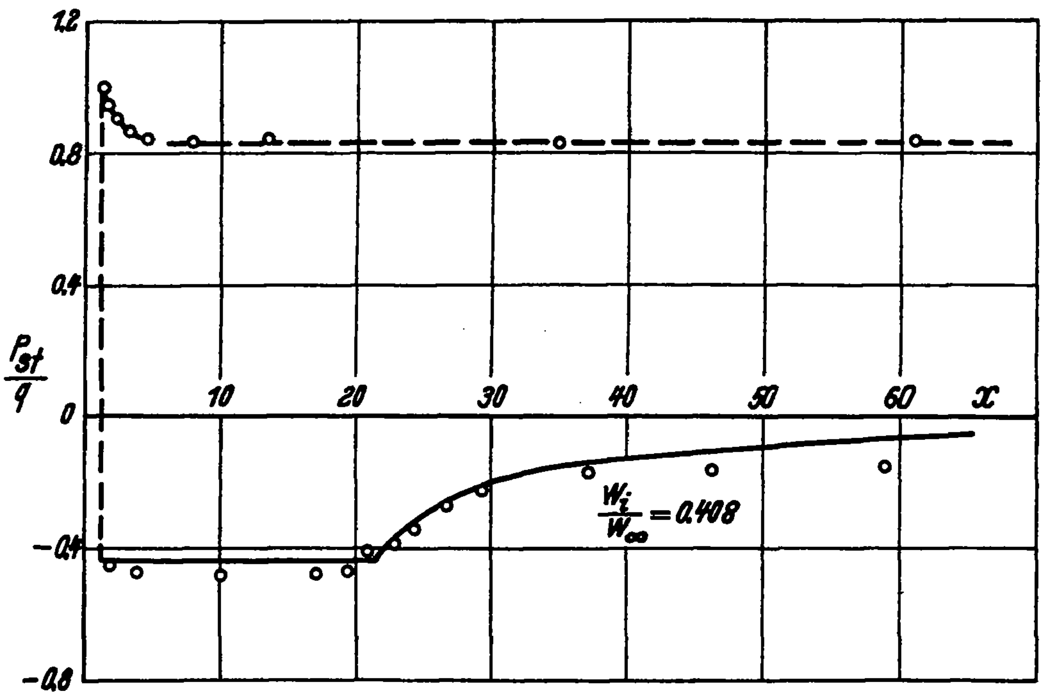


Figure 12.

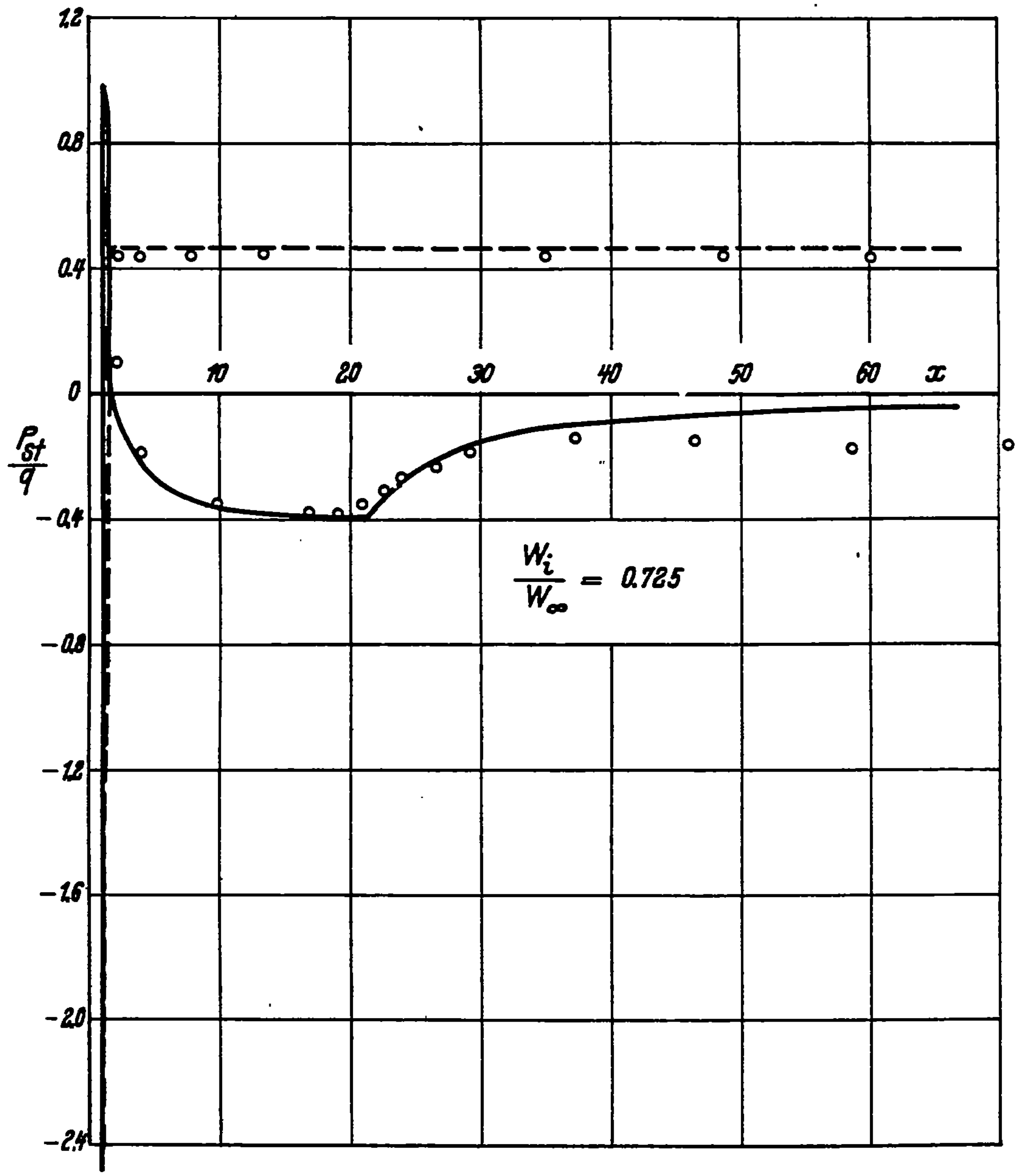

1.2
0.8
0.4
0
-0.4
-0.8
-1.2
-1.6
-2.0
-2.4
10
20
30
40
50
60
x
$\frac{P_{st}}{q}$
$\frac{W_i}{W_\infty} = 0.725$

Figure 13.

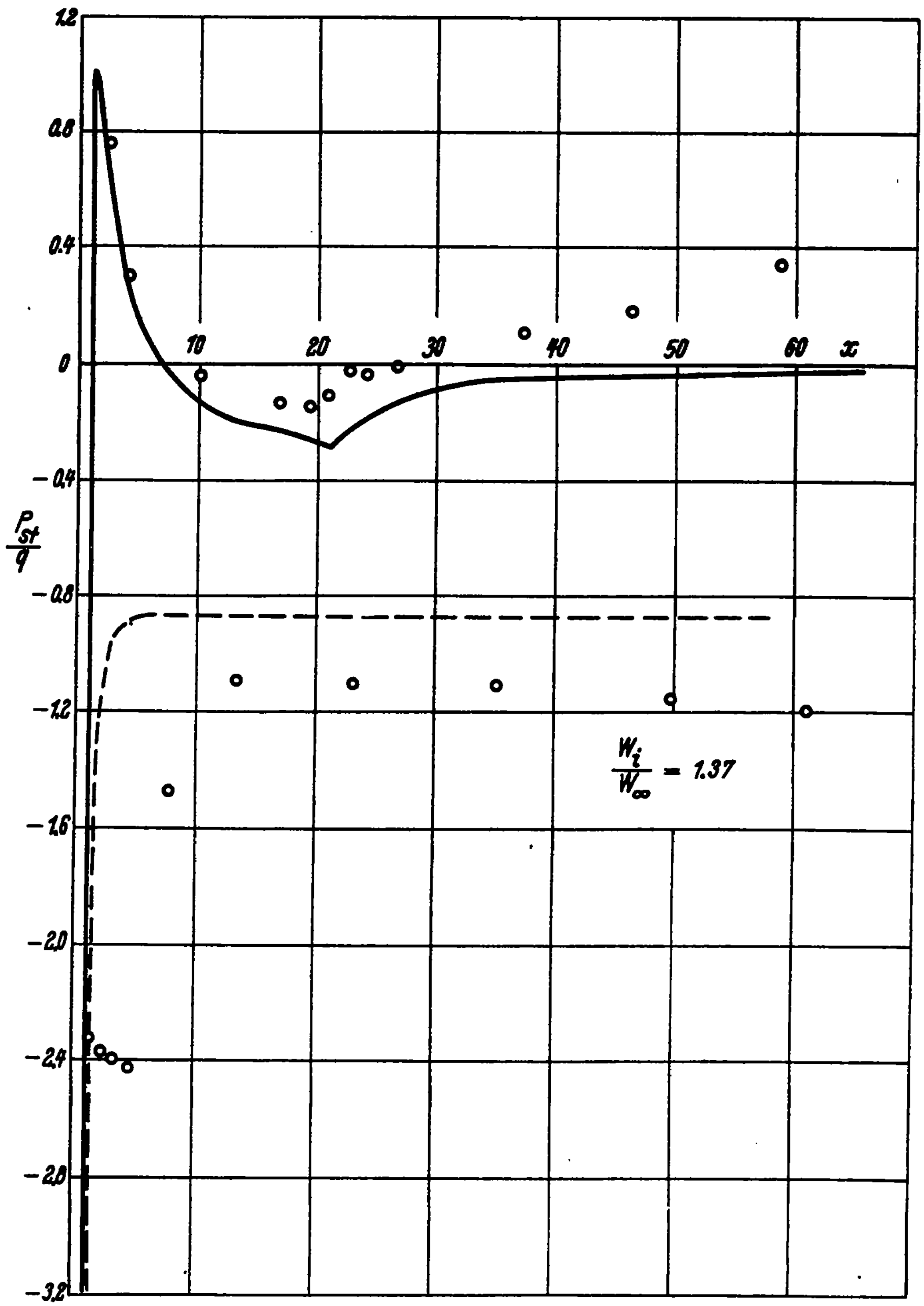
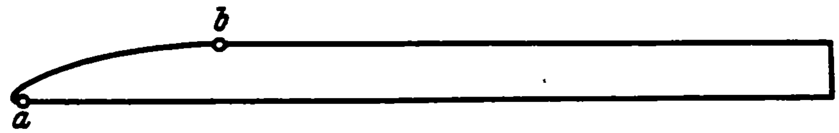

1.2
0.8
0.4
0
−0.4
−0.8
−1.2
−1.6
−2.0
−2.4
−2.8
−3.2
$\frac{P_{st}}{q}$
10
20
30
40
50
60
x
$\frac{W_i}{W_\infty}$ = 1.37

Figure 14.



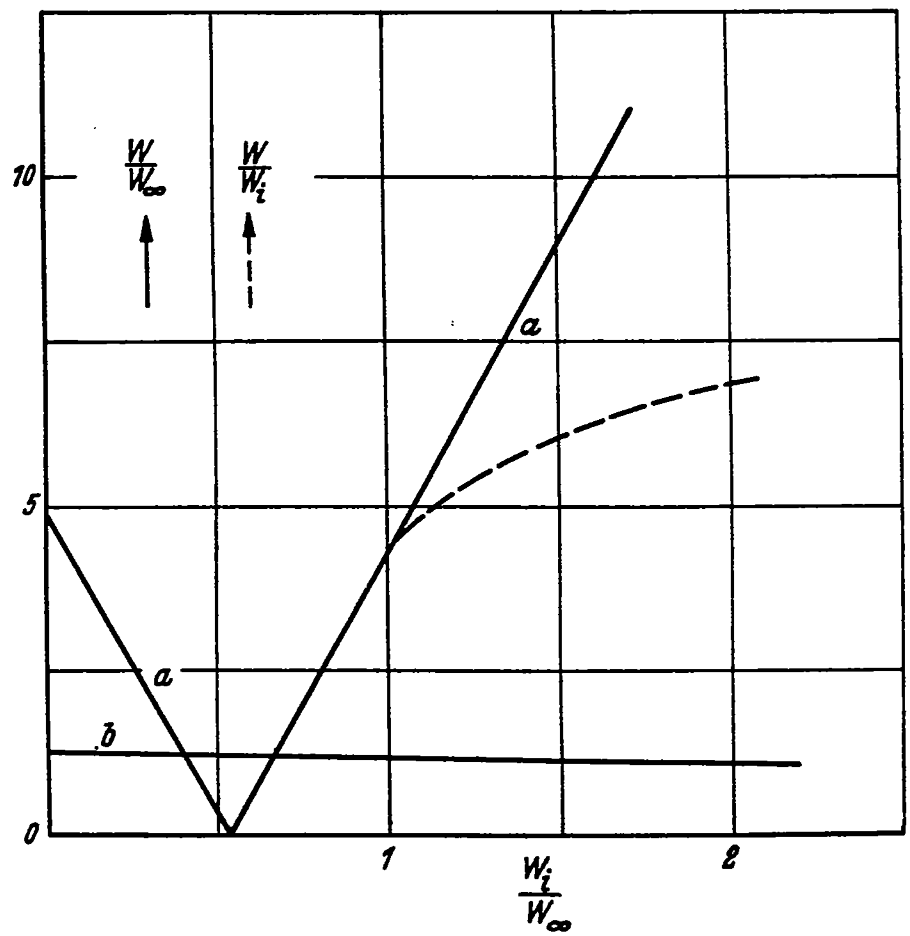


Figure 15.- The velocities at the end points of the nose of Ruden's inlet diffuser II as functions of w_i/w_∞. At the top is shown one half of the diffuser with points a and b.

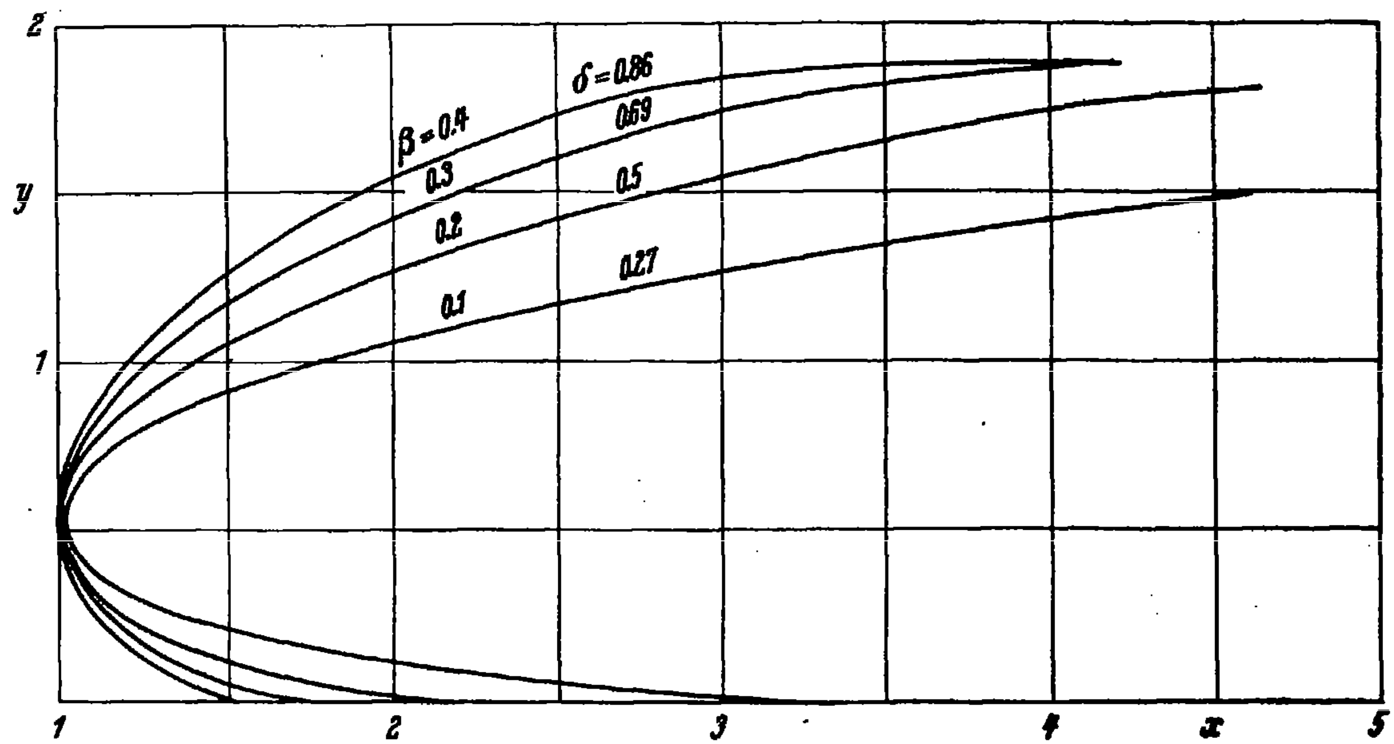


Figure 16.- Inlet diffusers of the third type for various values of the parameter β. $t_1 = 0.127$; $t_2 = 7.873$.

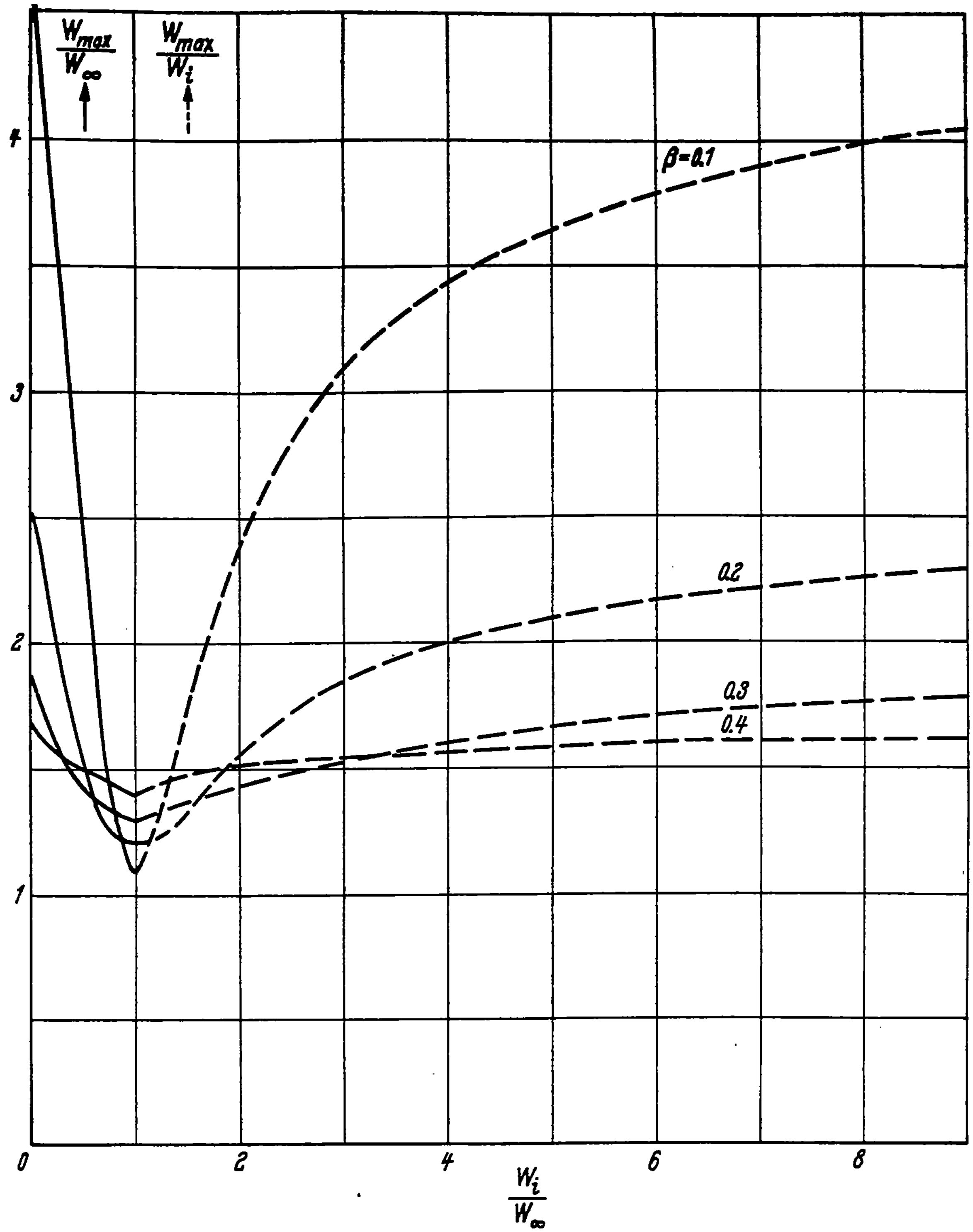
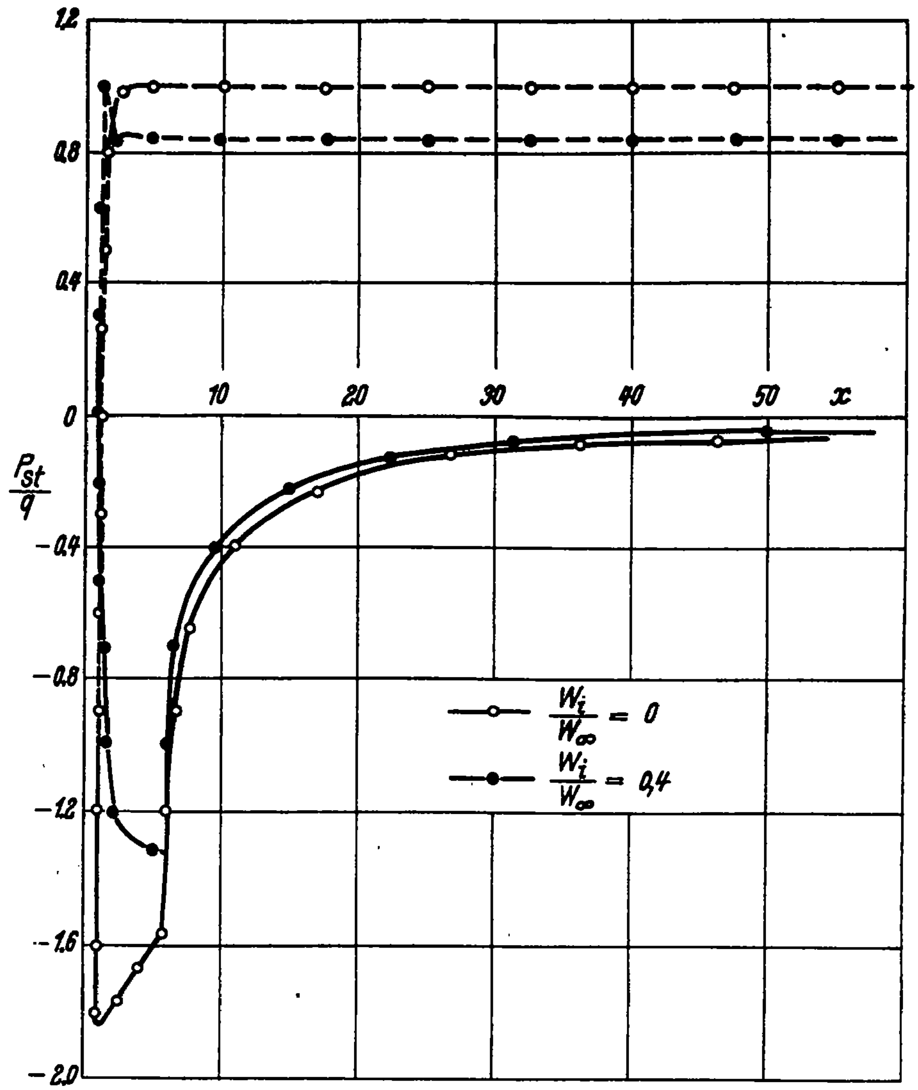


Figure 17.- Maximum velocities for the diffusers shown in figure 16.



Figures 18-20.- Pressure-distribution curves for the inlet diffuser $\beta = 0.4$ of figure 16.

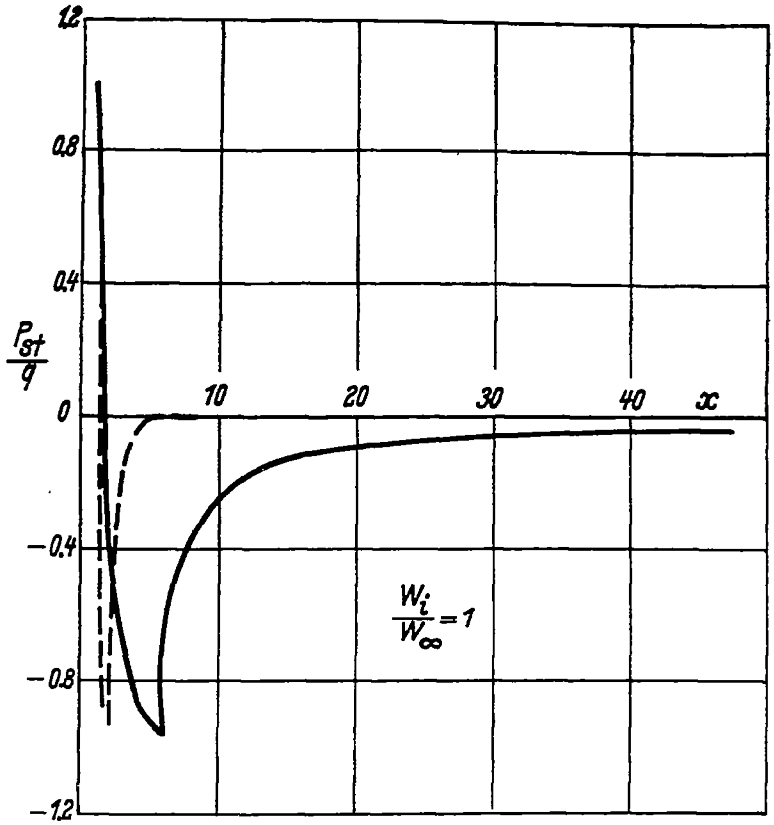


Figure 19.

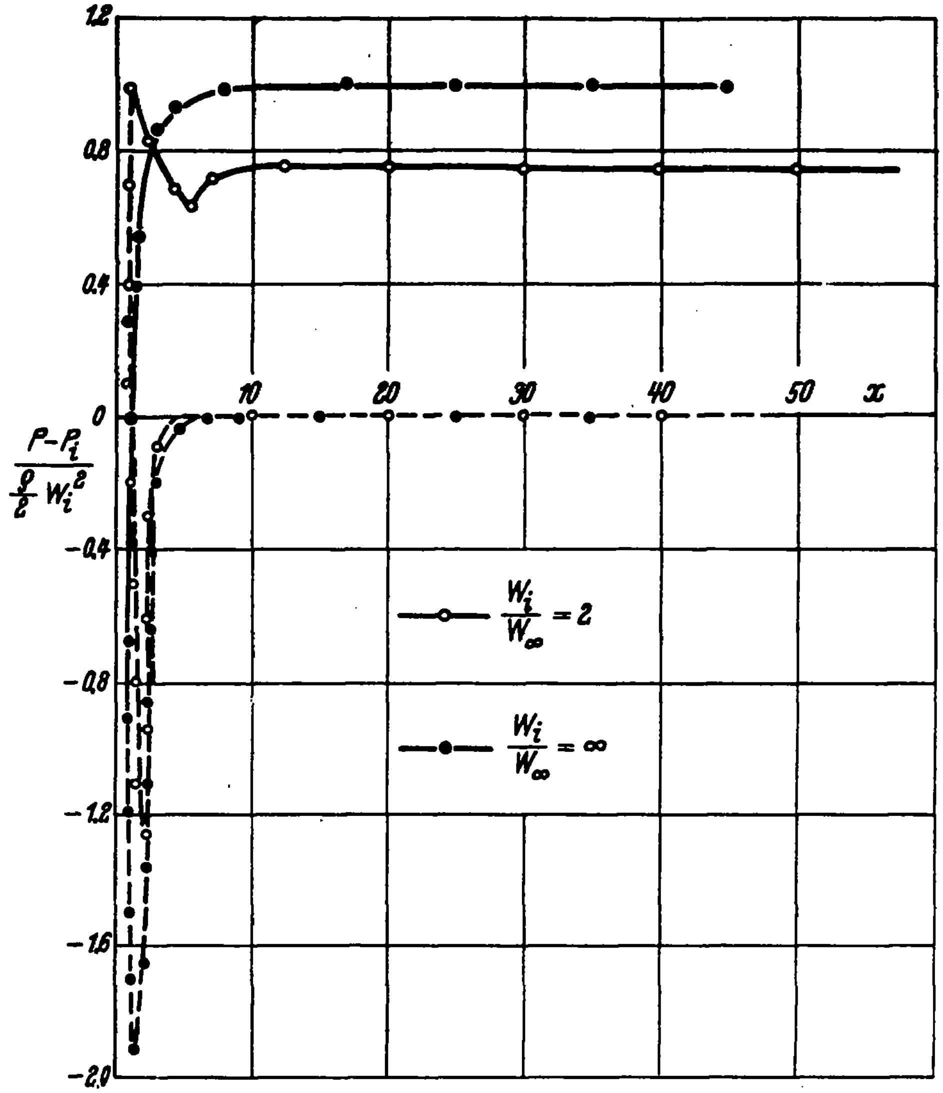


Figure 20.

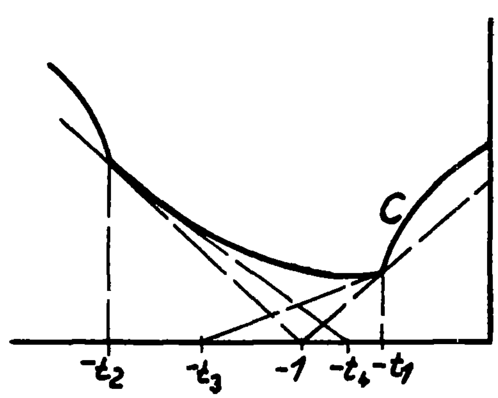


Figure 21.- For calculation of w_{max}.

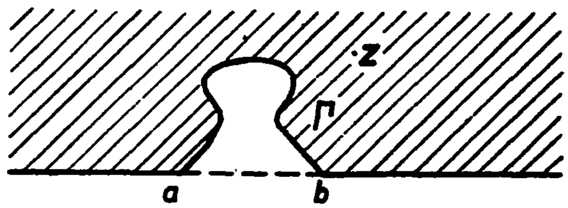


Figure 22.

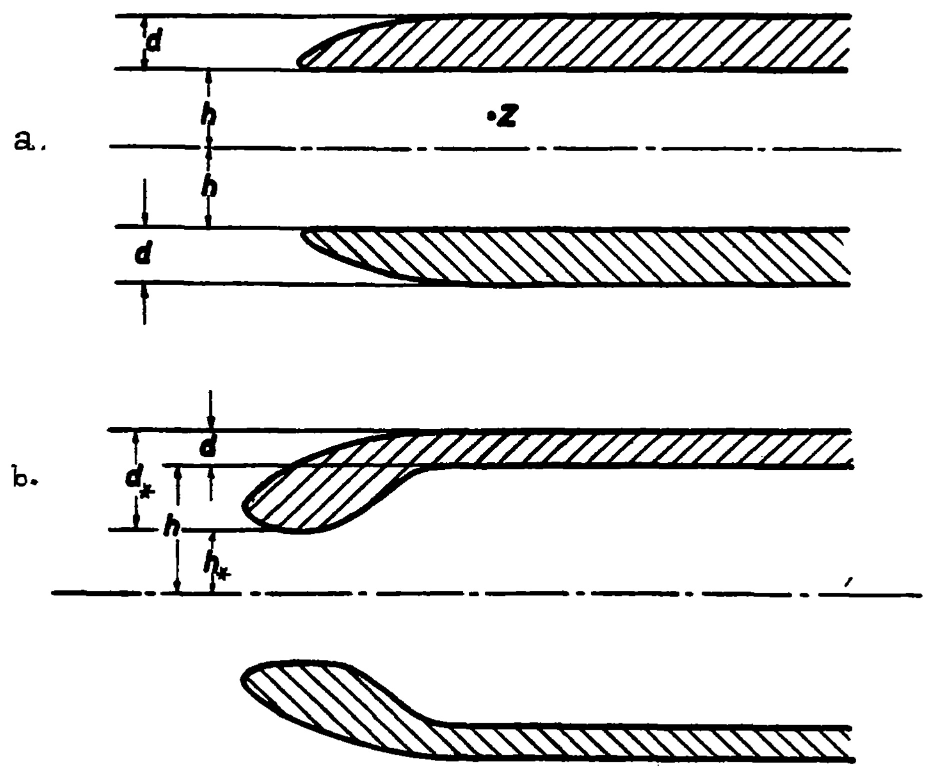


Figure 23.

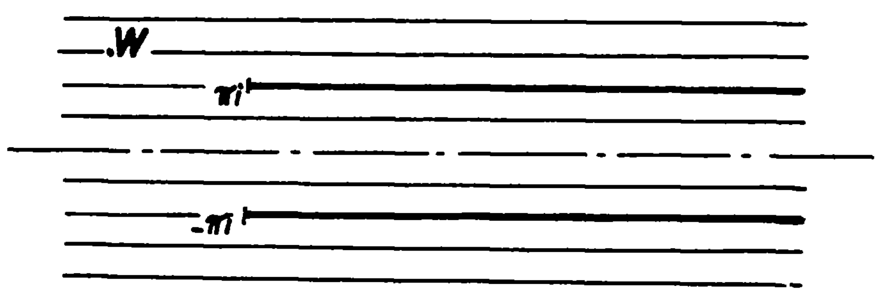


Figure 24.

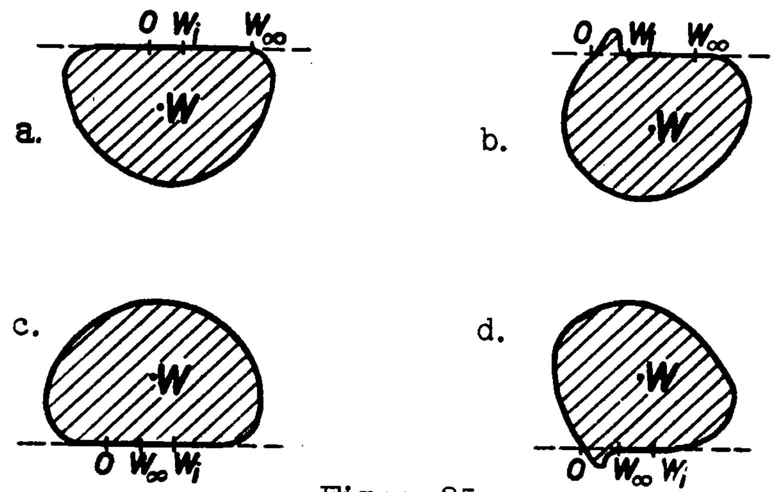
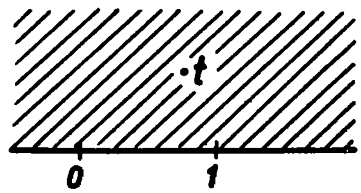


Figure 25.

Figure 26.



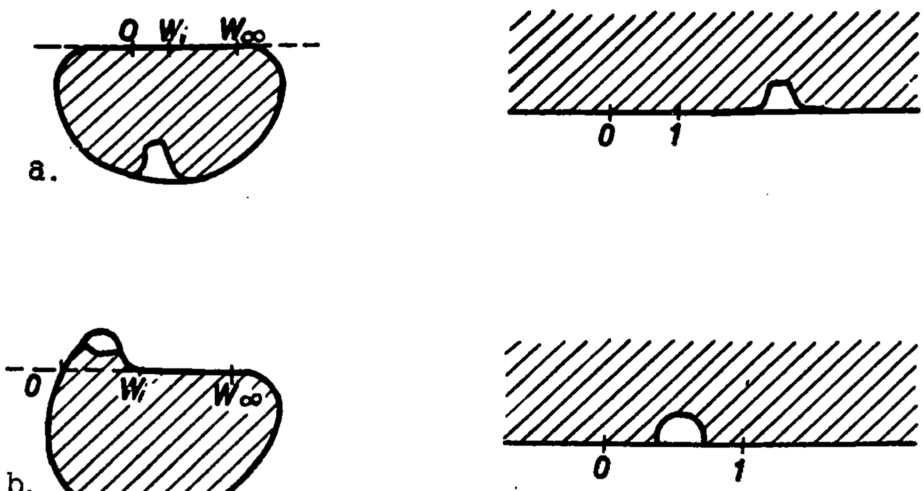


Figure 27.

Figure 28.